# Rail Transportation Models for Rural Populations

**By**

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# Rail Transportation Models for Rural Populations

Final Report - MBTC DOT 3024


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**Abstract**

Population growth in rural areas has led to new interest in rail transportation. Planning a passenger rail system involves numerous difficult decisions, most representing a trade-off between customer service and cost. In this work, we attempt to integrate many of these planning decisions. We consider strategic decisions such as station location and vehicle procurement, as well as tactical issues that include vehicle scheduling. Our integrated model exploits the linear network structure that best suits many rural American communities, including Northwest Arkansas. Due to the intractability of the integrated rail planning problem, we have developed a customized heuristic approach to solve real world instances. In our case study, we have applied our model and solution methodology to study the possibility of implementing a passenger rail system in Northwest Arkansas. Our work represents the first steps in a passenger rail feasibility study for Northwest Arkansas, while providing new mathematical modeling and solution methodology contributions to the area of transportation research.

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# 1 Introduction

Rising fuel prices (see Figure 1) and growing populations in rural areas have led to interest in rail transportation as an environmentally conscious alternative to highway expansion for the alleviation of traffic congestion. Northwest Arkansas (NWA) is a prime example of this phenomenon. In fact, NWA was the sixth-fastest growing metropolitan area from 1990-2000 with a growth rate of 47.5% [26]. Though the growth rate has decreased slightly since 2000, the NWA population could surpass 1 million within 25 years if current growth rates continue [1]. Further evidence suggesting NWA as a natural candidate for passenger rail is the advantageous distribution of the area's population. Furthermore, a study by the University of Arkansas Community Design Center [7] points out that two-thirds of all current NWA residents live within one mile of existing rail right-of-way.

Passenger rail systems of differing sizes and capabilities are available to city planners. Common amongst alternatives are Light Rail, Heavy Rail and Commuter rail systems. According to the American Public Transportation Association (APTA), Light Rail systems (also known as streetcar, tramway, or trolley systems) typically feature electrically driven vehicles with power drawn from an overhead electric line. The APTA defines Heavy Rail systems (also known as metros or subways) to be those operating on an electric railway with the capacity for heavy volume of traffic. Finally, the APTA states that Commuter Rail systems are usually located along routes of current or former freight railroad, that their trains may be electric or diesel driven, and that they typically connect a metropolitan area to its suburbs [15]. The methods we describe in this work could be applied to any of these system types. However, given the existing rail right-of-way through the heart of the area, the Commuter Rail model is most applicable to Northwest Arkansas.

In addition to rail systems differing by type and purpose, the configuration of any system plays a key role in determining its operational capabilities and challenges. Figure 2 depicts two such passenger rail configurations. Radial networks are common in urban settings, where populations are spread over vast areas, and throughout Europe and Asia, where rail systems connect cities in all directions. Because of the complexity

[1] Recent growth rates based on data from the U.S. Census Bureau 2012 Statistical Abstract [5].

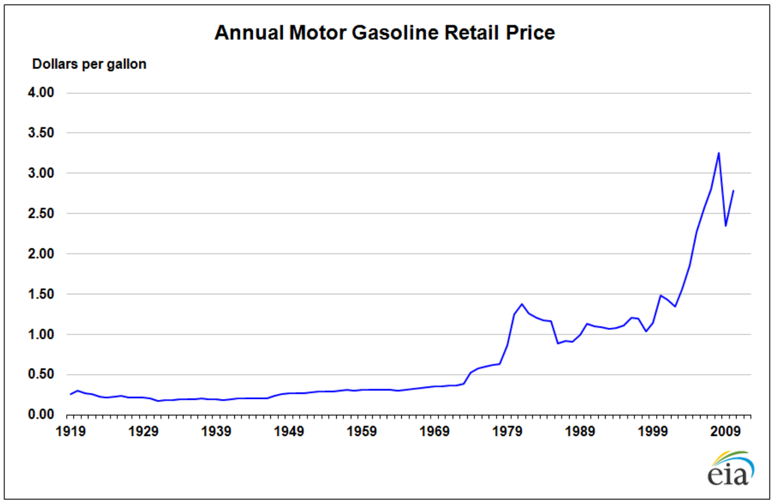
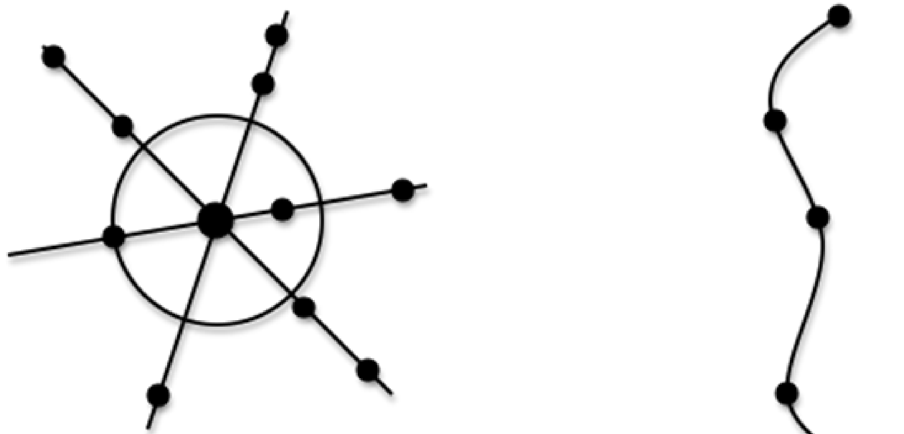



Figure 1: Historical Gasoline Prices in the U.S.
Source: U.S. Energy Information Administration

of most urban/regional radial rail networks, researchers have historically approached the rail planning problem hierarchically for the sake of tractability [10]. Any passenger rail system requires an extensive planning process that includes strategic, tactical, and operational decisions. Operational decisions typically concern day-to-day activities and schedule disruptions, tactical decisions are those with a 1-5 year impact (i.e. resource allocation), and strategic decisions are those with implications reaching beyond 5 years (i.e. resource procurement) [10]. Within each of these planning stages, numerous problems must be considered, as shown in Table 1. Authorities in the field have commented that the hierarchical planning approach fails to guarantee an optimal system, due to its inability to capture all interactions between various planning stages [9].

Some rural communities, especially those that have developed along a river, roadway, or historical rail line, lend themselves to the development of a passenger rail system that follows a single path, however. We will refer to these as linear networks (see Figure 2). Simpler networks with fewer required decisions may allow for the use of an alternative integrated planning process that simultaneously considers the set of all required decisions, yielding system-optimal solutions. Rural areas that naturally permit a linear network are prime candidates for this type of approach. In this work, we introduce a mixed integer programming model that integrates many of the strategic and tactical decisions outlined in Table 1. Since this integrated problem is difficult to solve



(a) Radial Network (b) Linear Network

Figure 2: Radial versus Linear network

Table 1: Rail Planning Decisions

| | |
|---|---|
| **Strategic** | Number of Stations |
| | Station Locations |
| | Track Location |
| | Number of Vehicles |
| **Tactical** | Days & Times of Operation |
| | Vehicle Routes |
| | Projected Demand |
| | Vehicle Schedules |
| **Operational** | Crew Composition |
| | Crew Assignment |
| | Train Dispatching |
| | Delay Management |

using currently available computer hardware and software, we have developed a customized heuristic process to generate quality solutions for realistically-sized instances. Finally, we have applied our model and solution methods to study the possibility of implementing a passenger rail system in Northwest Arkansas.

# 2 Literature review

Rail planning has been studied extensively within the operations research community. However, the existing literature is limited in its applicability to rural settings. The method adopted by most researchers consists of separating rail planning decisions into subproblems and solving each individually [10]. The following well-studied subproblems result: network planning [23], line planning [9, 16], station location [24, 27], timetabling [11, 20, 21], vehicle scheduling [28], and vehicle routing [30]. Though many of these problems are studied from a deterministic standpoint, some researchers have developed models that incorporate the uncertainty involved in rail systems. For example, Kroon et al. develop train timetables that minimize the average delay associated with stochastic disturbances in [21] and [20]. In [22], List et al. consider uncertainty of future demand and operating conditions in their model meant to optimize fleet sizes. Researchers have attempted to solve these problems exactly in rare cases. In [13, 14] a modified branch-and-bound technique is employed to solve certain rail and bus scheduling problems. The authors exploit the structure of the problems LP in a way that would not extend to our integrated problem, however. Much of the literature focuses on heuristic development since these problems are often applied to very large systems. Various heuristic approaches have been applied to these problems including Lagrangian Relaxation [8, 25], Tabu Search [17, 18, 25], Neighborhood Searches [18, 25], and Genetic Algorithms [17, 18]. In these works, heuristics have been shown to be successful in generating quality solutions for many rail planning problems. No single heuristic has been applied to all of the problems that we have integrated, however. For an extended review of passenger rail research see [10, 12]. More recent research shows a continued interest in this area, but no serious work has been done to integrate these various subproblems. Instead, researchers have continued to assume that the rail planning process will follow a hierarchical structure. This hierarchical approach to rail planning is unavoidable when the network to be constructed is more complex, as is the case with most Asian and European networks. However, due to subproblem interactions, it is in the planners' best interest to integrate decisions when possible [9]. Our investigation found that integration of this type is not present in the existing literature. However, we contend that the configuration and reduced size of many rural communities, including the NWA region, open the door to a partial integration of the rail planning process. In this work, we have taken the first steps toward an improved planning tool for rural rail transportation over the methods currently found in the literature. We have integrated many of the problems outlined above by exploiting the linear network structure that is best-suited for many rural settings. Since this integrated problem is difficult to solve to optimality, we have developed a heuristic motivated by the neighborhood search concept. Neighborhood search heuristics and their variants are ubiquitous in the Operations Research literature. For a review of local search techniques, see [29].

# 3 Problem description

Our model is meant to assist planners as they make important strategic and tactical decisions about potential passenger rail systems. We assume that a rail right-of-way has been determined, and that a finite number of potential station locations have been identified along this right-of-way. This right-of-way features one track for each direction of travel, and forms a *linear network* as defined in Section 1. Two of the potential stations form the static end-points for the potential system. One endpoint serves as the depot for the trains, where all trains begin and end each of their *loops* (see definition below). The opposite endpoint serves as the location where trains reverse their direction of travel.

**Loop** When a train departs the first station, traverses the entire track in one direction, travels the entire track in the opposite direction, and then returns to its original location, we will say that it has completed one loop.

Furthermore, we assume that origin, destination, and scheduling information is known or can be estimated for all potential customers. In this initial work, we have assumed deterministic customer demand. However, possibilities for stochastic variants of our

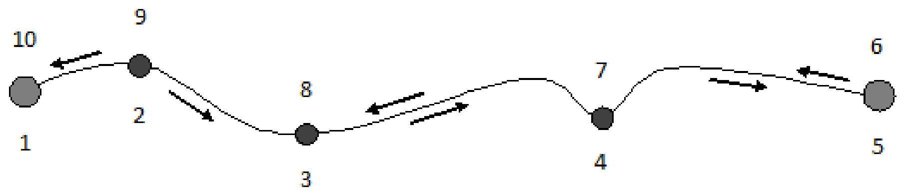



Figure 3: An Example Where $|\mathcal{L}| = 10$

problem are discussed in Section 7. Using this information, along with station and vehicle cost information, we have developed a model to identify the station configuration, vehicle fleet size, and set of train schedules that will maximize the daily profit for the system. We focus on a single-day horizon, with time measured in minutes. To normalize costs, daily values for item procurement costs are estimated (in current dollars) using the item's purchase cost and estimated life of the item. For example, if it costs \$2,000,000 to procure a train that should remain in service for 20 years for 250 working days each year, the estimated daily cost for each train is \$400. Costs associated with installing any necessary track are not considered, because the right-of-way is assumed to exist in our scenario.

To model our problem, we consider a passenger rail network consisting of two parallel tracks, one for each direction of travel, connecting two fixed stations (endpoints of the linear network) where trains turn around and depart in the opposite direction. The set $\mathcal{L}$ contains two elements for each possible station location along the track, corresponding to the two directions of travel. Letting $L = |\mathcal{L}|$, station 1 and $L$ both correspond to the first possible location, 2 and $L-1$ to the second possible location, and so on until the last possible station location, represented by $\frac{L}{2}$ and $\frac{L}{2}+1$. The cost of procuring a station at location $\ell \in \mathcal{L}$ is $f_\ell$, where $f_\ell > 0$ for $l = 1 \ldots \frac{L}{2}$ and 0 otherwise. Figure 3 provides an illustrative example. In this example, five potential stations have been identified, including the two fixed endpoints. Therefore, $L = 10$ and the first station location is represented by 1 and 10 depending on direction of travel, the second by 2 and 9, and so on.

The set of trains that may potentially serve customers is denoted by $\mathcal{T}$, where $|\mathcal{T}| = T$. Associated with each train $\tau \in \mathcal{T}$ is a procurement cost $c_\mathcal{T}$, a per-loop operating cost $v_\tau$, and a capacity $u_\tau$. The time required for a train to travel from location $\ell - 1$ to $\ell$ is denoted $t_\ell$. The speed of a train as it travels between locations is assumed to be constant, thus $t_\ell$ is proportional to the length of the track between $\ell - 1$ and $\ell$. In addition, we must account for the time required for a train to stop at location $\ell$, denoted $\delta_\ell$, in the event that a station exists there. The set $\mathcal{K}$, where $|\mathcal{K}| = K$, consists of the loops that a train may take around the linear rail network. Here, $K$ is a calculated upper bound on the possible number of loops that any train may need to take during the time horizon.

The set $\mathcal{G}$ is comprised of the set of potential passenger groups in the rail system. The number of passengers in group $g \in \mathcal{G}$ is denoted by $P_g$. The origin and destination for group $g$ are denoted $o_g$ and $d_g$, respectively. Each passenger group $g$ has an arrival window $[a_g - b_g, a_g]$ associated with their destination, where $a_g$ specifies the latest acceptable arrival time at destination $d_g$, and $b_g$ specifies the maximum acceptable waiting time (where waiting occurs when a passenger arrives at their destination early). Thus, any train arriving at $d_g$ within the arrival window of group $g$ is eligible to serve all or some of the passengers in group $g$. We restrict passenger groups from being split between multiple trains. Finally, the system earns a daily revenue of $r_g$ for serving a passenger in group $g$. Our system is assumed to operate $H$ minutes per day.

The decision variables included in the model formulation representing our problem are defined in Table 2. The model follows.

Table 2: Decision variables

| Variable | Type | Description |
|---|---|---|
| $z_\ell$ | binary | 1 if station constructed at $\ell$, for $\ell = 1, \ldots, \frac{L}{2}$, 0 otw |
| $x^l_{g,\tau}$ | binary | 1 if group $g$ assigned to train $\tau$ on $k^{th}$ time $\tau$ visits $o_g$, 0 otw |
| $\rho^k_{g,\tau}$ | integer | number of customers from group $t$ served by train $\tau$ on $k^{th}$ loop |
| $y_\tau$ | binary | 1 if train $\tau$ is used, 0 otw |
| $q^k_\tau$ | binary | 1 if train $\tau$ in use for $k^{th}$ loop, 0 otw |
| $w^k_{l,\tau}$ | continuous, $\geq 0$ | time at which train $\tau$ arrives at $l$ for $k^{th}$ time |
| $n^k_{l,\tau}$ | integer | number of passengers on train $\tau$ leaving location $l$ for $k^{th}$ time |

$$\text{maximize} \sum_{k \in \mathcal{K}} \sum_{\tau \in \mathcal{T}} \sum_{g \in \mathcal{G}} r_g \rho^k_{g,\tau} - \sum_{\ell = 1 \ldots \frac{L}{2}} f_\ell z_\ell - \sum_{\tau \in \mathcal{T}} c_\tau y_\tau - \sum_{k \in \mathcal{K}} \sum_{\tau \in \mathcal{T}} v_\tau q^k_\tau$$

subject to

$$
\begin{aligned}
z_\ell &\leq z_1 && \ell = 2\ldots(L/2)-1 && (1)\\
z_\ell &\leq z_{\mathrm{L}/2} && \ell = 2\ldots(L/2)-1 && (2)\\
x^k_{g,\tau} &\leq z_{I(o_g)} && \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} && (3)\\
x^k_{g,\tau} &\leq z_{I(d_g)} && \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} && (4)\\
q^k_\tau &\leq y_\tau && \tau\in\mathcal{T}; k\in\mathcal{K} && (5)\\
y_{\tau+1} &\leq y_\tau && \tau\in\mathcal{T}\setminus\{T\} && (6)\\
x^k_{g,\tau} &\leq q^k_\tau && \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} && (7)\\
\rho^k_{g,\tau} &\leq P_g x^k_{g,\tau} && \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} && (8)\\
\sum_{\tau\in\mathcal{T}}\sum_{k\in\mathcal{K}} x^k_{g,\tau} &\leq 1 && g\in\mathcal{G} && (9)\\
w^{k+1}_{1,\tau} &= w^k_{\mathrm{L},\tau} && \tau\in\mathcal{T}; k\in\mathcal{K}\setminus\{K\} && (10)\\
w^k_{\ell+1,\tau} &= w^k_{\ell,\tau} + \delta_\ell z_{I(\ell)} + t_{\ell+1} && \tau\in\mathcal{T}; k\in\mathcal{K}; \ell\in\mathcal{L}\setminus\{L\}. && (11)\\
(a_g-b_g)x^k_{g,\tau} &\leq w^k_{\ell,\tau} && \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} && (12)\\
(W-a_g)(x^k_{g,\tau}-1) &\leq a_g - w^k_{\ell,\tau} && \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} && (13)\\
n^k_{\ell,\tau} - n^k_{\ell-1,\tau} &= \sum_{g\in\mathcal{G};o_g=\ell}\rho^k_{g,\tau} - \sum_{g\in G;d_g=\ell}\rho^k_{g,\tau} && \tau\in\mathcal{T}; k\in\mathcal{K}; \ell\in\mathcal{L}\setminus\{1\} && (14)\\
n^k_{1,\tau} &= \sum_{g\in\mathcal{G};o_g=1}\rho^k_{g,\tau} && \tau\in\mathcal{T}; k\in\mathcal{K} && (15)\\
n^k_{\ell,\tau} &\leq u_\tau && \tau\in\mathcal{T}; k\in\mathcal{K}; \ell\in\mathcal{L} && (16)\\
z_\ell &\in \{0,1\} && \ell = 1\ldots L/2 && (17)\\
x^k_{g,\tau} &\in \{0,1\} && \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} && (18)\\
\rho^k_{g,\tau} &\in \mathbb{Z}^+ && \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} && (19)\\
y_\tau &\in [0,1] && \tau\in\mathcal{T} && (20)\\
q^k_\tau &\in [0,1] && \tau\in\mathcal{T}; k\in\mathcal{K} && (21)\\
n^k_{\ell,\tau} &\geq 0 && \tau\in\mathcal{T}; k\in\mathcal{K}; \ell\in\mathcal{L} && (22)\\
w_\tau &\geq 0 && \tau\in\mathcal{T} && (23)
\end{aligned}
$$

where

$$I(\ell) = \begin{cases} \ell & \text{if } \ell \leq \frac{L}{2} \\ L - \ell + 1 & \text{otherwise.} \end{cases}$$

In this formulation, constraints (1) and (2) force stations to be opened at the first and last (physical) locations if stations are opened at any other locations. This does assume that planners know where the system must begin and end. Since the endpoints of a rail system serve as depots to store, maintain and repair trains, we do not treat the locations of these two important facilities as separate decisions in this work. Constraints (3) and (4) ensure that a group cannot be served unless a station exists at its origin location and its destination location. Constraints (5) enforce the relationship between the $q$ variables and the $y$ variable for each train. Constraints (6) break symmetry by forcing trains to be used in order. Constraints (7) prohibit the assignment of groups to inactive trains. Constraints (8) establish the relationship between the $x$ variables and the $\rho$ variables for each group. Constraints (9) ensure that groups are served by at most one train, on exactly one of its loops. Constraints (10) and (11) enforce the train schedule based on the station configuration. Constraints (12) and (13) enforce destination arrival windows for customer assignments. The constant $W$, used in Constraints (13) and defined as $W = K \sum_{\ell \in \mathcal{L}} (\delta_\ell + t_\ell) + H$, is a logical upper bound for $w_{\ell,\tau}^k$. Constraints (14)-(16) enforce the capacity limitation for each train as it departs each location on each of its loops. Finally, (17)-(23) define the decision variables. Note that variables $y$, $q$, and $n$ are continuous, but will take on integer values in any feasible solution due to the problem structure.

The $w$ variables in the above formulation make it simple to understand and model the movement of trains in the system. Due to the network structure assumed above, however, it is possible to eliminate many of the $w$ variables using a simple substitution. Once a train enters the system, its movement is implicitly controlled by the configuration of the stations and the time spent at each station. Therefore, a single variable $w_\tau$, representing the time train $\tau$ enters the system, can replace $w_{\ell,\tau}^k$ using the following substitution:

$$w_{\ell,\tau}^k = w_\tau + k \sum_{\ell' \in \mathcal{L}} (\delta_{\ell'} z_{I(\ell')} + t_{\ell'}) - \sum_{\ell' \in \mathcal{L}: \ell' > \ell} (\delta_{\ell'} z_{I(\ell')} + t_{\ell'}) - \delta_\ell z_{I(\ell)}. \tag{24}$$

We will adopt notation to simplify this substitution. By letting

$$S(\ell, k) = k \sum_{\ell' \in \mathcal{L}} (\delta_{\ell'} z_{I(\ell')} + t_{\ell'}) - \sum_{\ell' \in \mathcal{L}: \ell' > \ell} (\delta_{\ell'} z_{I(\ell')} + t_{\ell'}) - \delta_\ell z_{I(\ell)}, \tag{25}$$

the substitution becomes

$$w_{\ell,\tau}^k = w_\tau + S(\ell, k). \tag{26}$$

This substitution eliminates the need for constraints (10) and (11). This more compact representation will be used in the computational testing discussed in Section 5. However, before considering computational issues, we explore an alternative approach for efficiently generating solutions to our problem in the following section.

# 4 Solution methodology

Despite the linear structure of the rail network considered in this work, experimentation has shown that our integrated problem requires a prohibitive amount of time to solve using commercial optimization software. To illustrate this point, Table 3 reports *typical* computational times[2] for four instances generated randomly based on real data gathered during this project. Note that this table is strongly indicative of results seen across all experiments considered in this project. For more information on the construction of these instances, see Section 5.

Table 3: CPLEX Results

| Instance | $\lvert\mathcal{G}\rvert$ | $L$ | $T$ | $H$ | $K$ | Runtime (s) | Best Soln. | Best Bnd. | Opt. Gap |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 200 | 8 | 10 | 500 | 8 | 36,000 | 125 | 1305.05 | 944.04% |
| 2 | 300 | 14 | 20 | 500 | 6 | 36,000 | 2484 | 5890.27 | 137.13% |
| 3 | 1000 | 30 | 25 | 700 | 9 | -* | -* | -* | -* |
| 4 | 2500 | 44 | 40 | 900 | 10 | -* | -* | -* | -* |

* Indicates that the memory on our test machine was exhausted

As Table 3 shows, our problem is difficult to solve even for very small test instances and larger instances exhaust the available memory on our test machine very quickly.

[2]All experiments were performed using CPLEX 12 on an Apple® iMac ® computer with an Intel® Core 2 ™ 2.66 GHz processor and 4 GB of RAM.

Furthermore, we have found the problem to be difficult to solve even when many of the decisions are fixed. For example, Table 4 shows the computational results for our problem when the station procurement decisions and the vehicle procurement decisions have been fixed (i.e. we assume that we know which stations should be opened and which trains are utilized).

Table 4: CPLEX Results with Fixed Station Configuration and Train Procurement

| Instance | Runtime (s) | Best Soln. | Best Bnd. | Opt. Gap |
|---|---|---|---|---|
| 1 | 18000 | 239 | 1025.74 | 329.18% |
| 2 | 18000 | 4172 | 5039 | 20.78% |
| 3 | -* | -* | -* | -* |
| 4 | -* | -* | -* | -* |

* Indicates that the memory on our test machine was exhausted

Since CPLEX has such difficulty solving the train scheduling and customer assignment subproblems, it was apparent that any solution methodology for the problem should rely very little, if at all, on exact approaches to solve portions of the problem. With this in mind, and because none of the heuristic techniques present in the literature could be easily adapted to our model, we developed a customized heuristic solution methodology to solve the problem described in the previous section. An overview of this procedure is described in the remainder of this section. More information regarding the detailed mechanics of the heuristic can be found in the Appendix.

Our heuristic is neighborhood search-based, but features both segmented routines and a nested structure within each of these routines. To clarify, our heuristic follows the high-level process outlined in Figure 4, where portions of the best found solution are carried over between each segment. The routines included in Figure 4 are described as follows.

**Initial Solution Construction Routine**

In the Initial Construction and Improvement routine, a starting station configuration is generated by exhaustively considering all possible station configurations and choosing the station configuration that maximizes the "potential profit" associated with station costs and passenger revenue. That is, if a passenger's origin and destination locations each possess a station, then his ticket revenue will be counted towards the "potential profit." At this point, any costs associated with serving that passenger other than station construction are ignored (i.e. train procurement, operational costs). Once

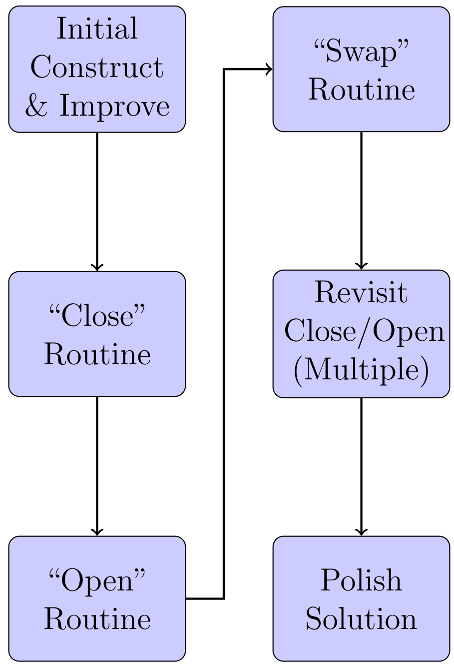



Figure 4: Heuristic Outline

a configuration has been identified, a greedy method is used to construct an initial solution. In this greedy method, each train is assigned a schedule that allows it to serve the largest unserved group at the time. Other unserved groups are added if this schedule allows them to be served. After all possible groups are added, the train is checked for profitability (i.e. Does the customer revenue on that train at least cover the costs associated with the train?). If the train is profitable, it is kept, otherwise its passengers are removed and the process starts over with the next-largest unserved group as a "seed." This repeats until all trains are active and profitable or until all unserved groups have been used as a schedule "seed." This solution is used to establish a baseline fleet size. Next, a user-defined range of fleet sizes, (number of trains) around this baseline will be considered. In our implementation, a fleet size range of 5 is used. For each fleet size in this range, a solution is greedily constructed that utilizes the specified number of vehicles (i.e. vehicles are given schedules and passengers are assigned to vehicles that could feasibly serve them). Finally, we attempt to improve this constructed solution by modifying the train schedules and customer assignments through the so-called improvement routine described below.

**Improvement Routine**

In the improvement routine, a solution obtained via a separate routine (e.g. Construction, Close, Open, etc.) is taken as an input and serves as the basis for our improvement procedure. Given the current solution, a user-defined number of train pairs are randomly selected, each train having an equal likelihood of being selected, for additional consideration. The number of train pairs allowed in our implementation is 200. Amongst the trains available for random selection is a dummy train to which all unserved customers are assigned. For each train pair considered, "group moves" and "group swaps" are attempted. A group move consists of moving a group served on one randomly selected train to the other randomly selected train. Similarly, a group swap consists of randomly choosing a group, where each group has an equal probability of being chosen, from each randomly chosen train and forcing each of these groups to be served by the train to which they are not currently assigned. In each of these cases, an attempt is made to serve the group or groups in question on the opposite train by allowing the number of people served within a group to be modified in order to satisfy train capacity. Next, an attempt is made to alter the train schedule so that any newly considered customers might be served. This step is done via a straightforward modification of the train arrival windows. However, it is important to note that while the train schedules can be altered to serve a new customer, no currently served customers may become unserved in this process. The sequence of group moves and swaps are repeated until a user-defined number of iterations have passed without improvement, at which point the next train pair is considered and the process is repeated. In our implementation, the limit on consecutive iterations without improvement is taken to be 25. The improvement routine stops when all randomly selected train pairs have been considered.

**Close Routine**

The Close routine begins with the station configuration identified in the Initial Solution Construction. From this configuration, we explore a portion of the neighborhood of solutions defined to be those with one fewer station than the current best solution. In the first Close routine iteration, we consider all station configurations achieved by closing exactly one station that is open in the current candidate solution. For each of these new configurations, the process outlined in the previous routines is performed (Configuration → Construct for Baseline Fleet Size → Fleet Size → Construct → Improve ). After this iteration, the solution with the highest objective among those considered is stored, even if it does not improve upon the overall best solution. This process is repeated with the stored solution's configuration (not necessary the configuration from the overall best solution) serving as the starting configuration for the next Close iteration. The iterations continue until no improvement has occurred for a user-defined number of Close iterations, or until all stations are closed. In our implementation the allowed number of iterations without an improvement is 2 for this routine.

**Open Routine**

A similar process as that found the Close routine is used to define Open routine. In this case, we partially explore the neighborhood of solutions that can be found by opening exactly one additional station in the configuration associated with the overall best solution found to this point. All configurations that feature one more open station than the current best are considered and the solution among these with the highest objective is stored as the input to the next Open iteration. This process repeats until a user-defined number of iterations have occurred with no improvement, or until all of the stations are open. In our implementation the allowed number of iterations without an improvement is 2 for this routine.

**Swap Routine**

In the swap routine, one currently opened station is closed, and one currently closed station is opened. We start with the configuration that produced the overall best solution up to this point. All possible pairs of stations consisting of one open and one closed station are considered and follow the same process to produce and improve solutions as detailed in the Improvement routine. After all possible pairs are considered, the best of these solutions is stored and its configuration is the starting point for the next Swap iteration. This process repeats until a user-defined number of *consecutive* iterations pass without obtaining and improved solution. In our implementation the allowed number of consecutive iterations without an improvement is 15 for this routine.

**Modified Close/Open Routine**

In this routine, we revisit the Close and Open concept using a modified search scheme. Specifically, in an attempt to uncover complex interactions between multiple stations, we now allow up to 5[3] stations to be opened or closed at once. Starting with the configuration from the best overall solution, we randomly decide whether to open or close stations, and how many. A user-defined number of these configurations are considered (i.e. solutions are created and improved for each configuration paired with a range of fleet sizes) and the best of these solutions is stored. Our implementation allows for 50 of these configurations to be considered. This process is repeated with the stored configuration as a starting point for the next iteration until a pre-set number of iterations are completed without improvement. For our implementation, we require improvement after at most 2 additional iterations in order to continue this routine.

**Polishing Routine**

In our final routine, we attempt to polish the best solution found using the previous sequence of routines. In this segment of the heuristic, we no longer consider configuration changes, but instead focus on the train schedules and passenger assignments. We do this by performing an extended version of the schedule improvement process outlined above and by removing passengers from trains and train loops with very light ridership. That is, if the passengers assigned to a train loop do not cover the operational cost of that loop ($v_\tau$) or if the total ridership for a train does not cover the procurement and operational costs for that train, then the customers are removed from the loop or train, respectively. An attempt is made to serve these customers on other trains if train passenger capacity is available and train schedules allow for the customers to travel within their desired time windows. However, it is important to note that the overall system profit will increase from removing customers that do not generate revenue exceeding a loop cost even if these customers remain unserved.

Notice that each routine found in Figure 4, except the final solution polishing, follows the general steps outlined in Figure 5 in some way. The nested structure of the heuristic follows the natural hierarchy of rail planning decisions outlined above. Our heuristic, like our model, considers the interaction between these decisions in a way that is not currently present in the literature. Again, a more detailed outline of the heuristic process is given in Appendix B.

[3]The limit of 5 was put in place due to the increased computational cost incurred by considering multiple stations beyond this level.

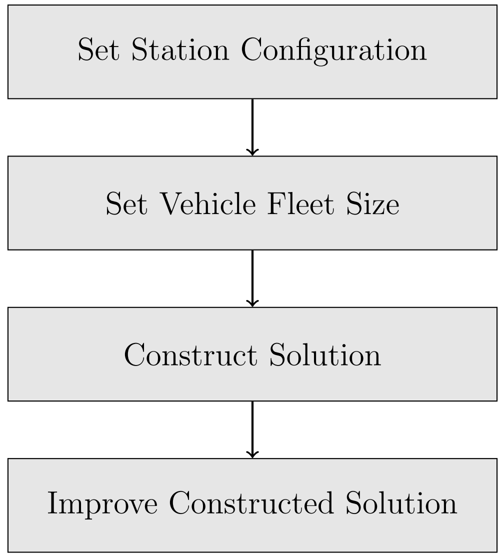



Figure 5: Nested Heuristic Structure

# 5 Computational study

In this section we investigate the performance of the solution methodology presented in Section 4 on a broad range of test instances. Before presenting our results, we discuss motivation and mechanics behind the scheme utilized to generate each of our random instances. Then we provide numerical results that offer insights into the capabilities of the proposed heuristic procedure.

## 5.1 Experimental design

Using information regarding existing rail systems and commuter patterns, we developed a procedure for randomly generating experiments intended to resemble real-world instances. The generation of our instances can be broken into: (i) determining potential station locations, (ii) assigning passenger demand time windows, (iii) identifying each potential customer's origin and destination and (iv) defining the appropriate rail system operational parameters. The following subsections describe how we handle the generation of each of these problem components.

**Station Locations**

The potential station locations were randomly generated in a manner consistent with the variation of available locations in Northwest Arkansas, where we hope to apply our model. That is, we assume an ordered number of possible locations along a single line. Note that the line might be representative of an existing rail bed, as is the case in Northwest Arkansas. For each consecutive pair of potential station locations, the number of miles between each location is a uniform random variable with range $[0.5, \beta]$ miles, where $\beta \in [5, 15]$ miles, depending on the instance.

**Passenger Time Windows**

The time in which passenger demand occurs was generated in such a way that two distinct "peaks" were present in the planning horizon in order to represent "rush hour" demand caused by commuter traffic to and from work. Specifically, we generate customer demand so that 60-80% of demand occurs during two "peak" periods during the horizon. For a full-day horizon, these "peaks" may fall between 7:00-9:00 AM and 4:00-6:00 PM, for example. Note that for the instances studied in this section, the horizon length is 500 minutes, leaving the full-length horizon to be studied in Section 6. Therefore, accurately modeling peaks and valleys in customer demand throughout this horizon was accomplished by classifying our customers as one of three types: commuters, students, or others. Then, arrival deadlines were generated according to a uniform distribution bounded by the preset windows shown in Table 5 that are specific to each customer type. Recall that each customer has both an originating and return trip, therefore separate bounds are given for each trip. In addition, since customers classified as "others" are assumed to travel anytime throughout the day with equal likelihood, the deadline associated with this category's outgoing trip is generated uniformly between time 0 and the end of the horizon (500 minutes). The deadline for the return trip of the "other" customers occurs with equal likelihood at anytime between the deadline of the originating trip, which we refer to as $\bar{o}$, and the end of the horizon. Passenger arrival window lengths were equally likely to be 15 or 30 minutes

Table 5: Customer Arrival Deadlines By Type

| Customer Type/Trip Classification | Arrival Deadline Bounds (min) |
|---|---|
| Commuters Outgoing Trip | [100,175] |
| Commuters Return Trip | [375,450] |
| Students Outgoing Trip | [100,175] |
| Students Return Trip | [375,450] |
| Others Outgoing Trip | [0,500] |
| Others Return Trip | [$\bar{o}$,500] |

for some instances and 30 or 60 minutes for others. This was done simply to account for the various levels of flexibility that passengers might have.

**Passenger Origins and Destinations**

Passenger origins and destination were determined in a uniform manner (i.e. stations being equally likely to be an origin or destination) in some cases and with higher variability (i.e. stations weighted based on popularity) in others in order to model differing population and attraction distributions. To model increased variability in customer/destination location, we again relied on the characterization of customer demand as being baseline, variable or more variable. In the baseline case, customers were randomly assigned an origin station[4] from the set of randomly generated potential station locations, followed by a destination randomly chosen from the locations down-track of the assigned origin station. In the variable and more variable scenarios, the likelihoods of locations being chosen as destinations were varied with increasing intensity as we moved from the variable to more variable scenarios. Specifically, in the variable instances, the overall likelihood that a station serves as a customer origin or destination ranges between 7% and 23%, with the sum of the probabilities associated with each potential station is 1. For more variable instances, a wider range of 6-30% of customer demand per station is possible. Here again, the sum of the probabilities associated with each potential station must be 1. This scheme allowed us to represent the realistic situation in which one location is primarily residential and would be a likely customer origin in the morning whereas another location may be located in an industrial area where many customers work, but few live. Therefore, morning traffic would be heavy outbound from the residential area and inbound to the industrial area, while afternoon demand would follow the opposite pattern.

**Rail Operations**

With regard to more specific operations of the rail system, we assumed a cruising train speed of 35 miles per hour based on various existing rail systems found in the literature [1]. Furthermore, we assumed train costs between \$500,000 and \$2 million and 15-20 year operational lives depending on the instance. These values are consistent with the values presented in [1]. For randomly generated instances, station procurement costs were set between \$2.5 million and \$15 million with an assumed life of 50 years.

[4]each origin was equally likely

These values are slightly lower than many station costs for existing systems [1, 2] since we assume that stations in rural settings would be cheaper due to lower construction and real estate costs.

## 5.2 Numerical results

In this section, we provide a comparison of the performance of the heuristic approach described in Section 4 and that of a commercial optimization solver. All experiments were performed using CPLEX 12 on an Apple® iMac® computer with an Intel® Core 2™ 2.66 GHz processor and 4 GB of RAM. All instances considered in this section feature 14 locations, 20 potential trains, and were generated using the procedures and definitions presented in Section 5.1. Unless indicated otherwise, daily station costs for these instances are uniformly generated between \$200 and \$900 and the customer arrival window length, $b$, is selected to be 30 or 60 minutes with equal probability. The default capacity for trains in these instances is 175 passengers unless otherwise specified in the naming convention presented in the following subsection.

### 5.2.1 Instance naming convention

To describe the characteristics of each instance tested, a four-part naming convention is adopted. The computational instances are named according to the following convention.

- *First Character of Instance Name:* All instances begin with the letter B, V, or M. These letters are used to indicate baseline, variable, or more variable customer demand variability, respectively. Each of these levels were described in Section 5.1 and reflect the likelihood that a particular station is chosen as a customer's origin/destination.
- *Second Character of Instance Name:* The second character in all instance names is either a 3 or a 5. A value of 3 (5) indicates that there were 300 (500) groups considered in that specific instance.
- *Third Character of Instance Name (optional):* In some instances, we've investigated the impact of varying certain default parameters (e.g. station cost, train capacity or passenger arrival window). For instances in which this additional con-

sideration was made, a “special” character appears as the third character in the instance name. The optional third character may be an S, C or W, indicating a change in station cost, train capacity or passenger arrival windows, respectively. The details of the changes associated with each of these special characters is as follows:

  – S indicates that, on average, the station costs considered in that instance are higher than those considered in a default instance. Specifically, the station costs are now generated uniformly between \$500 and \$1000 dollars.
  – C indicates that the train capacity for that instance was reduced from the default of 175 passengers per train. The actual value of the train capacity considered in that instance is indicated as a superscript of the character C.
  – W indicates that passenger arrival windows are shorter than the default of 30 or 60 minutes. For that instance, the passenger arrival windows are taken to either be 15 or 30 minutes.

- *Last Characters of Instance Name (optional):* For certain combinations of problem characteristics, multiple random instances were generated. In these cases, different instances generated using the same parameter values are differentiated by an underscore ( _ ) followed by a replicate number.

To illustrate this naming convention, consider an instance named V5W_2. From the name, we know that the instance has variable customer demand with 500 groups and a reduced customer window length. The “_2” indicates that this is the second instance of type V5W. In the following section, we analyze the computational performance of our approach on instances identified using this naming convention.

### 5.2.2 Heuristic versus CPLEX

The instances presented in Table 6 were randomly generated simply to assess the value of our heuristic versus that found by CPLEX. Note that in each of these instances the number of loops, $K$, is set to the value of 5, with the remaining parameters identified by the naming convention and default parameter values discussed in Section 5.2.1. In the results associated with each of the 8 instances shown in Table 6, the heuristic obtains a solution with an objective notably better than that obtained by CPLEX in

Table 6: Comparison Between Heuristic and CPLEX

| Instance | Heur. Runtime (s) | Final Heur. Obj. | CPLEX Runtime (s) | Best CPLEX Obj. |
|---|---|---|---|---|
| V3_1 | 804 | 4195 | 7200 | 0 |
| V3_2 | 345 | 3149 | 7200 | 0 |
| V3_3 | 119 | 4943 | 7200 | 0 |
| B3 | 826 | 4059 | 7200 | 425 |
| B5 | 251 | 9847 | 7200 | 0 |
| V5_1 | 1404 | 7043 | 7200 | 0 |
| V5_2 | 108 | 9844 | 7200 | 0 |
| V5_3 | 1388 | 4105 | 7200 | 0 |

the allotted time of 2 hours. In fact, in 7 of the 8 instances, CPLEX failed to find a solution that improved upon a plan that "does nothing" (i.e. open no stations, serve no customers). The ability of our heuristic to obtain improved solutions over a commercial solver is amplified by the fact that CPLEX was given 2 hours to provide its solutions, while the heuristic, on average, required only 655.6 seconds. These results were very much indicative of all our attempts to use commercial optimization software to identify solutions to the rail planning problem. Consistently, the commercial optimization tool failed to obtain a solution that located any stations or served any customers. Fortunately, the heuristic approach described in Section 4 provided profitable solutions in each of these cases. However, further analysis is needed to assess the value of the proposed heuristic approach. Therefore, a broader set of tests will be considered in the next section that will assist us in understanding the effectiveness of the different phases of our heuristic in obtaining improved solutions to our problem.

### 5.2.3 Heuristic performance

A more comprehensive set of results used to assess the performance of our heuristic are presented through the expanded set of instances shown in Tables 7 and 8 below. Recall from Section 4 that our heuristic proceeds through three phases: (i) Construction; (ii) Improvement and (iii) Polishing. Table 7 serves to evaluate the impact of these three phases. The percentage shown in the Improvement column is calculated as follows:

$$\frac{z^{Final} - z^{Construction}}{z^{Construction}}.$$

Improvements indicated by a '-' indicate that the original construction solution was no better than the "do nothing" option. Therefore, while the final polished objective has

Table 7: Heuristic Results: Objective and Improvement

| Instance | Runtime (s) | Init. Constructed Obj. | Pre-Polish Obj. | Final Obj. | Improvement |
|---|---|---|---|---|---|
| B3_1 | 58 | 1995 | 2972.5 | 2977.5 | 49.25% |
| B3_2 | 502 | 2730 | 3087.5 | 3102.5 | 13.64% |
| B5_1 | 111 | 5972 | 7639.5 | 7727 | 29.39% |
| B5_2 | 204 | 5867 | 6869.5 | 6872 | 17.13% |
| B3S | 139 | -1177 | 0 | 0 | - |
| B5S | 292 | 2173 | 3673 | 3733 | 71.79% |
| V3_1 | 318 | -842 | 273 | 320.5 | - |
| V3_2 | 180 | 1588.5 | 2043.5 | 2066 | 30.06% |
| V5_1 | 259 | 2072.5 | 4180 | 4290 | 107.00% |
| V5_2 | 517 | 3864.5 | 6017 | 6182 | 60.00% |
| V3S | 635 | 229 | 1311.5 | 1311.5 | 472.71% |
| V5S | 1419 | 2776.5 | 4346.5 | 4369 | 57.36% |
| M3_1 | 214 | -222 | 1122.5 | 1132.5 | - |
| M3_2 | 910 | 1165 | 3239 | 3354 | 187.90% |
| M5_1 | 563 | 2623.5 | 4921 | 5173.5 | 97.20% |
| M5_2 | 305 | 2181.5 | 5973 | 6138 | 181.37% |
| M3S | 799 | 292.5 | 1640 | 1720 | 488.03% |
| M5S | 711 | 3002.5 | 4637.5 | 4682.5 | 55.95% |
| $B3C^{50}$ | 272 | -2069 | 0 | 0 | - |
| $B3C^{75}$ | 194 | -1683.5 | 0 | 0 | - |
| $V3C^{50}$ | 533 | -1444 | 0 | 0 | - |
| $V3C^{75}$ | 915 | -46.5 | 696 | 971 | - |
| $M3C^{50}$ | 458 | -1792 | 304.5 | 514.5 | - |
| $M3C^{75}$ | 951 | -611 | 1557.5 | 1690 | - |
| B3W_1 | 183 | -1755.5 | 0 | 0 | - |
| B3W_2 | 457 | 0 | 0 | 0 | - |
| V3W_1 | 341 | -1552.5 | 0 | 0 | - |
| V3W_2 | 422 | -483 | 0 | 0 | - |
| M3W_1 | 1284 | 519 | 1715 | 1772.5 | 241.52% |
| M3W_2 | 254 | -665.5 | 174.5 | 174.5 | - |

improved, the percent of improvement cannot be compared with the results in which the initial construction heuristic had a positive objective.

It is clear from Table 7 that the improvement and polishing phases have a significant impact on solution quality. On average, the percent improvement of the final objective over the initially constructed objective is 135.02% . All of these improved solutions were obtained in less than 30 minutes, with the average time required being 480 seconds. For the case in which default train capacity and passenger arrival window values (i.e. 175 passengers and arrival windows of 30 or 60 minutes) were used, our heuristic finds a profitable solution in 17 of the 18 corresponding instances. However, the results also suggest that decreasing the train capacity results in instances for which the heuristic has difficulty finding a solution that serves any customer demand profitably. A similar observation can made when the passenger arrival windows are reduced in the last 6 instances of Table 7. In this case, the heuristic fails to find a profitable solution in 4 out of the 6 considered instances.

Table 8 provides further insights into the actual characteristics of the final rail system plan obtained for each instance considered. We can see that in a majority of instances, approximately half of the stations considered for the system are actually chosen. Interestingly, if stations were opened, a high percentage of customers were served. Specifically, for instances in which at least one station was opened, over 84% of groups, as well as over 84% of total customer demand is satisfied. To serve these customers, only a relatively small fleet of trains was required as only 3-5 of the 20 potential trains were actually utilized in the final solutions.

Table 8: Heuristic Results: Final Solution

| Instance | Stations Open | Fleet Size | Groups Served | Passengers Served |
|---|---|---|---|---|
| B3_1 | 7 | 3 | 287/300 | 2973/3110 |
| B3_2 | 7 | 3 | 271/300 | 2925/3227 |
| B5_1 | 7 | 4 | 479/500 | 5002/5254 |
| B5_2 | 7 | 4 | 471/500 | 4540/4931 |
| B3S | 0 | 0 | 0/300 | 0/3123 |
| B56 | 7 | 4 | 455/500 | 4606/5135 |
| V3_1 | 5 | 4 | 229/300 | 2269/3111 |
| V3_2 | 7 | 3 | 228/300 | 2870/3134 |
| V5_1 | 7 | 4 | 422/500 | 4388/5309 |
| V5_2 | 7 | 3 | 432/500 | 4432/5266 |
| V3S | 5 | 3 | 230/300 | 2337/3188 |
| V5S | 7 | 4 | 424/500 | 4334/5237 |
| M3_1 | 6 | 3 | 242/300 | 2497/3127 |
| M3_2 | 5 | 3 | 274/300 | 2956/3248 |
| M5_1 | 7 | 4 | 427/500 | 4449/5264 |
| M5_2 | 5 | 3 | 464/500 | 4868/5281 |
| M3S | 7 | 4 | 289/300 | 3080/3240 |
| M5S | 7 | 5 | 444/500 | 4545/5284 |
| $B3C^{50}$ | 0 | 0 | 0/300 | 0/2905 |
| $B3C^{75}$ | 0 | 0 | 0/300 | 0/3244 |
| $V3C^{50}$ | 0 | 0 | 0/300 | 0/3213 |
| $V3C^{75}$ | 7 | 5 | 254/300 | 2664/3247 |
| $M3C^{50}$ | 3 | 4 | 171/300 | 1741/3063 |
| $M3C^{75}$ | 5 | 4 | 216/300 | 2278/3353 |
| B3W_1 | 0 | 0 | 0/300 | 0/3062 |
| B3W_2 | 0 | 0 | 0/300 | 0/3283 |
| V3W_1 | 0 | 0 | 0/300 | 0/3134 |
| V3W_2 | 0 | 0 | 0/300 | 0/3063 |
| M3W_1 | 6 | 5 | 286/300 | 3409/3583 |
| M3W_2 | 7 | 4 | 193/300 | 2179/3233 |

Unfortunately, we are unable to comment on the performance of our heuristic in relation to the optimal solution for any non-trivial instances since we have been unable to prove optimality for any such instance to date.

## 6 Case study: Northwest Arkansas

The University of Arkansas Community Design Center (CDC) publication, *Visioning Rail Transit in Northwest Arkansas* [7] is credited with the original inspiration for this work. In the study, city planners and University of Arkansas students made a case for rail transit in the Northwest Arkansas (NWA) region, which includes communities such as Fayetteville, Lowell, Springdale, Rogers, and Bentonville. The arguments presented in the report for developing a rail system in NWA were primarily qualitative in nature. Analytical planning tools were not used to select the station locations that were proposed, illustrated in Figure 6.

Northwest Arkansas is home to multiple post-secondary schools, including the University of Arkansas, attended by over 20,000 graduate and undergraduate students. Numerous companies also have their headquarters or regional offices in NWA, including Walmart Stores Inc., J.B. Hunt Transport, Inc. and Tyson Foods, Inc. Due to heavy inter- and intra-city commuting to these and other destinations, traffic congestion is an ever-present issue in the region. Rail transportation is one option available to city planners hoping to reduce congestion.

Using actual NWA commuter, geographical, and real estate data, along with information gleaned from existing rail systems in the U.S., we constructed an instance for the rail planning model that is representative of the NWA region. A formal feasibility study would require resources unavailable to us, but the instance we have generated mirrors the typical size and structure of any rail system that might be proposed in the area. In creating the instance, we explicitly accounted for different classes of passengers: (i) commuters, (ii) university passengers (e.g. students and teachers) and (iii) others (e.g. citizens going shopping or to visit others in the community). Instance details are given in Table 11 and described in the remainder of this section. Importantly, we show that the heuristic outlined in Section 4 is applicable to instances representative of growing rural areas and produces solutions in an acceptable amount of time, considering the integrated nature of our planning model. This is important because the NWA instance exhausts computer memory almost immediately when commercial

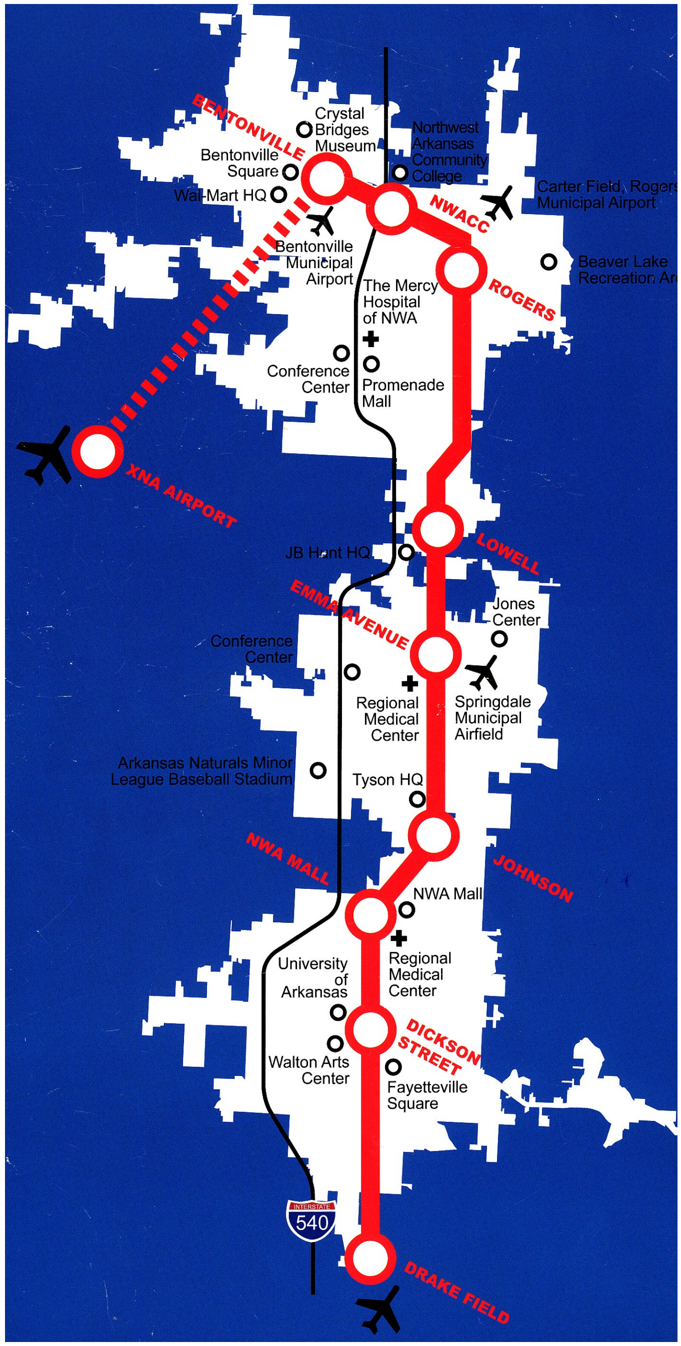



Figure 6: Map of NWA with CDC's Proposed System
Source: *Visioning Rail Transit in Northwest Arkansas* [7]

optimization software attempts to solve it.

A list of potential station locations were generated for the NWA instance utilizing our knowledge of the region, locations of large employers, residential areas, busy highways, schools, and other attractions. These locations, which all lie along an existing rail right-of-way, are listed in Table 9. The costs of building stations at these locations were estimated using publicly available assessed property values [3, 6] and costs for existing rail systems [1, 2]. We assume station costs in NWA would be lower than those in more urban areas due to the availability of lower cost land and labor, and our estimates reflect this assumption. Train procurement costs, operational costs, speed, and capacity were estimated using information available from existing systems [1]. We assume the time required for a train to stop at a station to unload and load passengers is 2 minutes and is independent of the station and number of passengers. A revenue per customer of $2.50 is assumed for all customers served regardless of their trip length. Trains are assumed to operate for 15 hours (900 minutes) per day, beginning at 6:00 am. To estimate zip code-to-zip code commuter volume, we used data from [4] compiled by [19] along with data from [26]. We assumed a 10% adoption level for most commuter lanes. This was reduced to 5% for passengers commuting between adjacent zip codes, and also for park-and-ride commuters. Adoption level was reduced to 1% for travel within a single zip code. Using data provided by the University of Arkansas and The Northwest Arkansas Community College (NWACC) outlining student commuter numbers by zip code, we used similar methods to estimate student demand. For each potential customer group, an original trip (usually morning) and a return trip were generated. If multiple potential stations existed in a single zip code, we assumed customers were equally likely to originate from any of these stations with two exceptions:

1. The station nearest a highway was selected as the origin for all park-and-ride customers.
2. Locations identified as light origins (e.g., Fayetteville Drake Field) were half as likely to serve as a customer origin than other locations in the same zip code.

When multiple potential stations existed within a zip code, destination selection was weighted by the size of employers located near the potential stations. For example, between the two stations in Bentonville, commuters were twice as likely to be destined for the Walmart HQ location over the alternate Bentonville location because Walmart employs a large percentage of Bentonville workers. Destinations were limited to school locations for students, and were randomly generated for other customers, though some locations were designated as attraction locations and more likely to be selected. Customer arrival deadlines were assigned uniformly within the ranges given in Table 10, and arrival window lengths were equally likely to be 30 or 60 minutes.

Table 9: NWA Instance Proposed Stations and Distances

| $\ell$ | Station Name | $\ell - 1$ to $\ell$ (miles) | Zip |
|---|---|---|---|
| 1 | Fay - Drake Field | 0 | 72701 |
| 2 | Fay - 15th St | 3.1 | 72701 |
| 3 | Fay - MLKJR Blvd | 0.7 | 72701 |
| 4 | Fay - Dickson St | 1.0 | 72701 |
| 5 | Fay - Maple Ave | 0.4 | 72701 |
| 6 | Fay - Cleveland St | 0.4 | 72701 |
| 7 | Fay - North St | 0.4 | 72703 |
| 8 | Fay - Sycamore St | 0.7 | 72703 |
| 9 | Fay - Township St | 0.9 | 72703 |
| 10 | Fay - Drake St | 0.6 | 72703 |
| 11 | Fay - Wash. Reg. Med. Cntr. | 0.5 | 72703 |
| 12 | Fay - Joyce St | 1.1 | 72704 |
| 13 | Johnson | 0.8 | 72704 |
| 14 | Spring - Tyson | 1.8 | 72762 |
| 15 | Spring - Robinson Ave | 1.5 | 72764 |
| 16 | Spring - Sunset Ave | 0.6 | 72764 |
| 17 | Spring - Emma Ave | 0.7 | 72764 |
| 18 | Lowell | 5.0 | 72745 |
| 19 | Rogers - New Hope Rd | 3.5 | 72758 |
| 20 | Rogers - Walnut St | 2.0 | 72756 |
| 21 | Benton - NWACC | 4.2 | 72712 |
| 22 | Benton - Walmart HQ | 2.6 | 72712 |

Table 12 outlines the results of our computational testing for the NWA instance. It demonstrates that our heuristic improved upon its initial constructed solution dramatically. Importantly, given that the objective of our problem is to maximize profit, it is interesting to learn that results *from this experiment* suggest that a profitable rail system may be attainable. Table 13 provides some interesting details regarding the characteristics of a possible NWA rail system. For the instance considered, the system would use 20 of the 22 possible station locations, omitting only the locations at Martin Luther King Blvd and Drake St in Fayetteville, AR. While we allowed for up to 40 trains to be included in the system, our solution suggests that only 7 trains are needed to attain a daily profit of \$3,645. Finally, it is interesting that almost 67% of potential

Table 10: NWA Parameter Details

| Parameter | Values |
|---|---|
| $a_g$ | 60-180 (morning commuters)<br>660-780 (evening commuters)<br>120-360 (morning students)<br>420-680 (evening students)<br>0-900 others |
| $b_g$ | 30/60 |
| $c_\tau$ | 290 |
| $f_\ell$ | 323-1933 (based on land price and station type) |
| $H$ | 900 |
| $P_g$ | 1-9 |
| $r_g$ | \$2.50 |
| $t_\ell$ | 0.7-8.6 (based on geography) |
| $u_\tau$ | \$150 |
| $v_\tau$ | \$100 |

Table 11: NWA Instance Overview

| | |
|---|---|
| Groups | 2830 |
| Total Potential Passengers | 14368 |
| Locations | 44 |
| Potential Trains | 40 |
| Horizon (min) | 900 |
| K | 10 |
| System Length (miles) | 32.5 |

passenger demand would be satisfied using the plan produced by the heuristic.

The strength of conclusions drawn from this experiment are limited by the strength of data available. Clearly, the approach proposed in this paper is most useful when reliable data obtained from a formal feasibility study is used to populate the model. Based on our investigation, there is evidence to suggest that a rail system is worth investigating further. In a future study conducted in partnership with city officials and financial experts, the heuristic described in this paper can effectively evaluate the feasibility of a NWA rail system.

Table 12: NWA Instance Computational Results

| | |
|---|---|
| Runtime (s) | 32334 |
| Initial Constructed Objective | -434.5 |
| Pre-Polish Objective | 2987.5 |
| Final Objective | 3645 |

Table 13: NWA Instance Final Solution Statistics

| | |
|---|---|
| Stations Open | 20/22 |
| Fleet Size | 7/40 |
| Groups Served | 1983/2830 |
| Passengers Served | 9620/14368 |

Table 14: NWA Instance Heuristic Solution Station Configuration

| $\ell$ | Station Name | Status |
|---|---|---|
| 1 | Fay - Drake Field | Open |
| 2 | Fay - 15th St | Open |
| 3 | Fay - MLKJR Blvd | Closed |
| 4 | Fay - Dickson St | Open |
| 5 | Fay - Maple Ave | Open |
| 6 | Fay - Cleveland St | Open |
| 7 | Fay - North St | Open |
| 8 | Fay - Sycamore St | Open |
| 9 | Fay - Township St | Open |
| 10 | Fay - Drake St | Closed |
| 11 | Fay - Wash. Reg. Med. Cntr. | Open |
| 12 | Fay - Joyce St | Open |
| 13 | Johnson | Open |
| 14 | Spring - Tyson | Open |
| 15 | Spring - Robinson Ave | Open |
| 16 | Spring - Sunset Ave | Open |
| 17 | Spring - Emma Ave | Open |
| 18 | Lowell | Open |
| 19 | Rogers - New Hope Rd | Open |
| 20 | Rogers - Walnut St | Open |
| 21 | Benton - NWACC | Open |
| 22 | Benton - Walmart HQ | Open |

# 7 Future work

Our efforts revealed a number of future research directions stemming from this work. First, we realize the importance of allowing trains to enter and leave the system to accommodate peaks in demand. This reduces costs by cutting out operational costs for some trains during periods with low demand. The model that we presented in this work does allow trains to "sit out" loops as a cost-saving measure, but the times that trains can possibly re-enter the system after leaving are mandated by the rigid train movement constraints. One resolution would be the development of an alternative model that requires a new non-negative continuous variable, $\Delta_\tau^k$, representing the delay time that train $\tau$ spends at the depot between loops $k-1$ and $k$ (defined for $k \in \mathcal{K} \setminus \{1\}$). This allows trains to delay for any non-negative amount of time between each loop.

These changes would result in the following model:

$$\text{maximize} \sum_{k\in\mathcal{K}}\sum_{\tau\in\mathcal{T}}\sum_{g\in\mathcal{G}} r_g \rho_{g,\tau}^k - \sum_{\ell\in\mathcal{L}} f_\ell z_{I(\ell)} - \sum_{\tau\in\mathcal{T}} c_\tau y_\tau - \sum_{k\in\mathcal{K}}\sum_{\tau\in\mathcal{T}} v_\tau q_\tau^k$$

subject to

$$x_{g,\tau}^k \le z_{I(o_g)} \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} \tag{27}$$

$$x_{g,\tau}^k \le z_{I(d_g)} \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} \tag{28}$$

$$z_\ell \le z_1 \qquad \ell = 2\ldots(L/2)-1 \tag{29}$$

$$z_\ell \le z_{\mathrm{L}/2} \qquad \ell = 2\ldots(L/2)-1 \tag{30}$$

$$q_\tau^k \le y_\tau \qquad \tau\in\mathcal{T}; k\in\mathcal{K} \tag{31}$$

$$y_{\tau+1} \le y_\tau \qquad \tau\in\mathcal{T}\setminus\{T\} \tag{32}$$

$$q_\tau^{k+1} \le q_\tau^k \qquad \tau\in\mathcal{T}; k\in\mathcal{K}\setminus\{K\} \tag{33}$$

$$x_{g,\tau}^k \le q_\tau^k \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} \tag{34}$$

$$\rho_{g,\tau}^k \le P_g x_{g,\tau}^k \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} \tag{35}$$

$$\sum_{\tau\in\mathcal{T}}\sum_{k\in\mathcal{K}} \rho_{g,\tau}^k \le P_g \qquad g\in\mathcal{G} \tag{36}$$

$$(a_g - b_g)x_{g,\tau}^k \le w_\tau + \bar{S}(d_g,k,\tau) \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} \tag{37}$$

$$(W - a_p)(x_{p,\tau}^k - 1) \le a_p - w_\tau - \bar{S}(d_g,k,\tau) \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} \tag{38}$$

$$\Delta_\tau^k \le H q_\tau^k \qquad \tau\in\mathcal{T}; k\in\mathcal{K}\setminus\{1\} \tag{39}$$

$$n_{\ell,\tau}^k - n_{\ell-1,\tau}^k = \sum_{g\in\mathcal{G};o_g=\ell} \rho_{g,\tau}^k - \sum_{g\in G;d_g=\ell} \rho_{g,\tau}^k \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; \ell\in\mathcal{L}\setminus\{1\} \tag{40}$$

$$n_{1,\tau}^k = \sum_{g\in\mathcal{G};o_g=1} \rho_{g,\tau}^k \qquad \tau\in\mathcal{T}; k\in\mathcal{K} \tag{41}$$

$$n_{\ell,\tau}^k \le u_\tau \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; \ell\in\mathcal{L} \tag{42}$$

$$z_\ell \in \{0,1\} \qquad \ell = 1\ldots L/2 \tag{43}$$

$$x_{g,\tau}^k \in \{0,1\} \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} \tag{44}$$

$$\rho_{g,\tau}^k \in \mathbb{Z}^+ \qquad \tau\in\mathcal{T}; k\in\mathcal{K}; g\in\mathcal{G} \tag{45}$$

$$y_\tau \in [0,1] \qquad \tau\in\mathcal{T} \tag{46}$$

$$q_\tau^k \in [0,1] \qquad \tau\in\mathcal{T}; k\in\mathcal{K} \tag{47}$$

$$n_{\ell,\tau}^{k} \geq 0 \qquad \tau \in \mathcal{T}; k \in \mathcal{K}; \ell \in \mathcal{L} \tag{48}$$

$$w_{\tau} \geq 0 \qquad \tau \in \mathcal{T} \tag{49}$$

$$\Delta_{\tau}^{k} \in [0, H] \qquad \tau \in \mathcal{T}; k \in \mathcal{K}\backslash\{1\} \tag{50}$$

where

$$\bar{S}(\ell, k, \tau) = S(\ell, k) + \sum_{2 \leq k' \leq k} \Delta_{\tau}^{k'}. \tag{51}$$

It would also be interesting to investigate how to allow customers to be served by multiple potential stations, while capturing their preference of one station over another. This is relevant when multiple "park & ride" facilities could serve an intermodal customer or in very densely populated areas where multiple potential station locations are being considered in a relatively small area.

Finally, the inclusion of the stochastic nature of passenger demand, train schedules, and/or future population growth is needed to assist planners in accounting for the uncertainty associated with community growth. We are especially interested in modeling the phenomenon that has been observed after many passenger rail systems are implemented in which the rail system itself causes a shift in population growth trends.

# 8 Conclusions

The model we have proposed is a first step at integrating many decisions faced by rail system planners. It is important to point out that we consider only a portion of the costs associated with an operational commuter rail system. In fact, we have focused on a subset of the overall process required to design a new rail system. We do feel, however, that we have integrated portions of the planning process that have typically been considered separately in the planning process.

Our heuristic is motivated by concepts that are mature in the operations research community. However, the neighborhood considered in our heuristic is unique from those considered in single-stage planning problems. The model that we developed is very complex, and the strong interaction between continuous and discrete decisions made it difficult to apply common heuristic methods without extensive customization.

In real-world applications, complex side constraints and interactions are common, and heuristic techniques are often necessary to produce suitable solutions. We feel that our customized approach can serve as to assist others faced with problems not easily solved by "out of the box" heuristics.

Finally, our case study provides a glimpse into the real world applicability of integrated rail planning models. Our results for NWA indicate that a passenger rail system may, in fact, be a good option for planners in the area to pursue further. Our objective value should not be interpreted to mean such a system would operate with large daily profits since many costs were not considered here (e.g. administrative overhead, track construction). We have merely shown that such a system could potentially be an operational success with modest adoption levels.

With further development, and with technological advances in computing, integrated methods should eventually become a reality for rail planners in rural settings and later in larger systems.

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# 9 Appendix A: Notation and decision variables

## Sets

Table 15: Sets

| | |
|---|---|
| $\mathcal{G}$ | $\mathcal{G}$, indexed by $g$, represents the set of potential passenger groups in the rail system (members of the same passenger group share origins, destinations, and arrival deadlines) |
| $\mathcal{K}$ | $\mathcal{K} = \{1 \ldots K\}$, indexed by $k$, represents the number of loops that a train may make around the network where $K$ is a calculated upper bound on the possible number of loops (A loop is defined to be leaving the first station and visiting each open station in the network before returning to the first station) |
| $\mathcal{L}$ | $\mathcal{L} = \{1 \ldots \lvert\mathcal{L}\rvert\}$, indexed by $\ell$, contains all possible station locations along the track with one element of $\mathcal{L}$ corresponding to each location for each direction of travel. That is, station 1 and $L$ both correspond to the first station, 2 and $\lvert\mathcal{L}\rvert - 1$ to the second possible station, and so on until the last station, which is represented by $\frac{\lvert\mathcal{L}\rvert}{2}$ and $\frac{\lvert\mathcal{L}\rvert}{2} + 1$. |
| $\mathcal{T}$ | $\mathcal{T} = \{1 \ldots T\}$, indexed by $\tau$, represents the set of trains that could potentially serve customers on the network |

# Parameters

Table 16: Parameters

| | |
|---|---|
| $a_g$ | The latest possible arrival time for group $g$ (defined for $g \in \mathcal{G}$, $0 \leq a_g \leq H$) |
| $b_g$ | The maximum amount of time that passenger group $g$ may arrive before its arrival deadline, $a_g$ (defined for $g \in \mathcal{G}$, any train that arrives at $d_g$ before $a_g$ but after $a_g - b_g$ is eligible to serve all or some of the passengers in group $g$ |
| $c_\tau$ | The cost of procuring train $\tau$ (defined for $\tau \in \mathcal{T}$) |
| $d_g$ | The destination location for group $g$ (defined for $g \in \mathcal{G}$, $d_g \in \mathcal{L}$) |
| $f_\ell$ | The cost of procuring a station at location $\ell \in \mathcal{L}$ (defined for $\ell = 1 \ldots L/2$) |
| $H$ | The length of the service horizon, in minutes |
| $o_g$ | The origin location for group $g$ (defined for $g \in \mathcal{G}$, $o_g \in \mathcal{L}$) |
| $P_g$ | The total number of passengers in group $g$ (defined for $g \in \mathcal{G}$) |
| $r_g$ | The revenue for serving one passenger from group $g$ a single time (defined for $g \in \mathcal{G}$) |
| $t_\ell$ | The time required for any train to traverse the distance from location $\ell - 1$ to location $\ell$ at cruise speed (defined for $\ell \in \mathcal{L}$, for $\ell = 1$ this value should be zero or should represent the delay between arriving at location $\lvert\mathcal{L}\rvert$ on loop $k$ and starting loop $k + 1$) |
| $u_\tau$ | The capacity of train $\tau$, in customers (defined for $\tau \in \mathcal{T}$) |
| $v_\tau$ | The cost of operating train $\tau$ for one loop (defined for $\tau \in \mathcal{T}$) |
| $W$ | $W = \lvert\mathcal{K}\rvert \sum_{\ell \in \mathcal{L}}(\delta_\ell + t_\ell) + H)$ is a logical upper bound for the $w^k_{\ell,\tau}$ variables |
| $\delta_\ell$ | The additional time required to stop at location $\ell$ in the even that a station exists there, including the delay associated with deceleration and acceleration (defined for $\ell \in \mathcal{L}$) |

# Decision Variables

Table 17: Decision Variables

| | |
|---|---|
| $n^k_{\ell,\tau}$ | A non-negative, continuous variable representing the number of passengers on train $\tau$ leaving location $\ell$ for the $k^{th}$ time (defined for $\ell \in \mathcal{L}; \tau \in \mathcal{T}; k \in \mathcal{K}$, may be a continuous nonnegative variable but will take on integer values due to problem structure) |
| $q^k_\tau$ | A continuous variable on $[0,1]$ that takes a value of 1 if train $\tau$ is active for loop $k$ and 0 otherwise (defined for $\tau \in \mathcal{T}; k \in \mathcal{K}$, may be continuous on $[0,1]$ but will take on integer values due to problem structure) |
| $w^k_{\ell,\tau}$ | A continuous non-negative variable that represents the time that has elapsed from the beginning of the horizon to the time that train $\tau$ arrives at location $\ell$ for the $k^{th}$ time (defined for $\ell \in \mathcal{L}; \tau \in \mathcal{T}; k \in \mathcal{K}$) |
| $w_\tau$ | After the variable-space reduction, the nonnegative continuous variable $w_\tau$ represents the time that train $\tau$ enters the system (defined for $\tau \in \mathcal{T}$) |
| $x^k_{g,\tau}$ | A binary variable with a value of 1 if group $g$ is assigned to train $\tau$ on its $k^{th}$ loop and 0 otherwise (defined for $g \in \mathcal{G}; \tau \in \mathcal{T}; k \in \mathcal{K}$) |
| $y_\tau$ | A continuous variable on $[0,1]$ that takes a value of 1 if train $\tau$ is active for any loop and 0 otherwise (defined for $\tau \in \mathcal{T}$, may be continuous on $[0,1]$ but will take on integer values due to problem structure) |
| $z_\ell$ | A binary variable with value 1 if a station is procured at location $\ell$ and 0 otherwise (defined for $\ell = 1 \ldots \lvert\mathcal{L}\rvert/2$) |
| $\Delta$ | If we add delay model to future work |

# 10 Appendix B: Detailed heuristic outline

The outline below describes our heuristic in moderate detail. Simple or self-explanatory functions have not been outlined completely, but some of the more complicated functions have been described below the heuristic. The file *instance.dat* contains values for all instance parameters. The file *parameters.txt* contains values for user-defined heuristic settings. At various points, solution information is saved in four different global "bins:" $cand, best, hold$, and $tsih$. These "bins" store some or all of the values for problem variables and the objective value.

```
MAIN(instance.dat, parameters.txt)
  // create necessary instance data structures and read values from instance.dat
  INITIALIZE(instance.dat)
  // read heuristic parameters from parameters.txt
  READPARAMS(parameters.txt)
  // return the station configuration with the greatest potential profit,
  // ignoring scheduling issues and train costs
  candZ = GETSTARTINGCONFIG()
  // construct fleet size and solution from scratch
  CONSTRUCTSOLUTION()
  // store new best solution
  REPLACEBESTWITHCAND()
  activeTrains = SUM(candY)
  holdObj = −10000
  for i = 0 to FLEETRANGE − 1
      fleetSize = activeTrains − FLOOR(FLEETRANGE/2) + i
      for j = 0 to 1
          // construct solution for specific fleet size using method j
          CONSTRUCTFLEETSOLUTION(fleetSize, j)
          if candObj > bestObj − TITHRESHOLD
              // improve cand solution by manipulation customer
              // assignments and train scheduling
              IMPROVECANDSOLUTION(fleetSize, TCUTOFF, GCUTOFF)
          if candObj > holdObj
              REPLACEHOLDWITHCAND()
  if holdObj > bestObj
      REPLACEBESTWITHHOLD()
```

Main Cont'd

```
stepZ = bestZ
holdObj = −10000
noImpr = 0
numOpen = Sum(bestZ)
for i = 0 to numOpen − 3
    candZ = stepZ
    for ℓ = 1 to L/2 − 2
        if candZ[ℓ] == 1
            candZ[ℓ] = 0
            ConstructSolution()
            activeTrains = Sum(candY)
            for i = 0 to fleetRange
                fleetSize = activeTrains − Floor(fleetRange/2) + i
                for j = 0 to 1
                    ConstructFleetSolution(fleetSize, j)
                    if candObj > bestObj − tiThreshold
                        ImproveCandSolution(fleetSize, tCutoff, gCutoff)
                    if candObj > holdObj
                        ReplaceHoldWithCand()

    stepZ = holdZ
    if holdObj > bestObj
        ReplaceBestWithHold()
        noImpr = 0
    else
        noImpr+ = 1
        if noImpr > closeCutoff
            Break
if bestObj < 0
    ReplaceBestWithZero()
stepZ = bestZ
holdObj = −10000
noImpr = 0
numClosed = L/2 − Sum(bestZ)
if numClosed == L/2
    stepZ[0] = 1
    stepZ[L/2-1] = 1
```

MAIN CONT'D

```
for i = 1 to numClosed − 3
    candZ = stepZ
    for ℓ = 1 to L/2 − 2
        if candZ[ℓ] == 0
            candZ[ℓ] = 1
            ConstructSolution()
            activeTrains = Sum(candY)
            for i = 0 to fleetRange − 1
                fleetSize = activeTrains − Floor(fleetRange/2) + i
                for j = 0 to 1
                    ConstructFleetSolution(fleetSize, j)
                    if candObj > bestObj − tiThreshold
                        ImproveCandSolution(fleetSize, tCutoff, gCutoff)
                    if candObj > holdObj
                        ReplaceHoldWithCand()
    stepZ = holdZ
    if holdObj > bestObj
        ReplaceBestWithHold()
        noImpr = 0
    else
        noImpr+ = 1
        if noImpr > openCutoff
            Break
stepZ = bestZ
holdObj = −10000
noImpr = 0
numOpen = Sum(bestZ)
numClosed = L/2 − numOpen
for i = 0 to swapSteps − 1
    for j = 0 to numClosed − 1
        for m = 0 to numOpen − 1
        // open j^th closed station and close m^th open station
        candZ = SwapOpenClosed(j, m)
        ConstructSolution()
        activeTrains = Sum(candY)
        for i = 0 to fleetRange − 1
            fleetSize = activeTrains − Floor(fleetRange/2) + i
            for j = 0 to 1
                ConstructFleetSolution(fleetSize, j)
                if candObj > bestObj − tiThreshold
                    ImproveCandSolution(fleetSize, tCutoff, gCutoff)
                if candObj > holdObj
                    ReplaceHoldWithCand()
```

```
        stepZ = holdZ
        if holdObj > bestObj
            ReplaceBestWithHold()
            noImpr = 0
        else
            noImpr+ = 1
            if noImpr > swapCutoff
                Break
    stepZ = bestZ
    holdObj = -10000
    noImpr = 0
    for i = 0 to revisitSteps - 1
        for j = 0 to revisitStepSize - 1
            // randomly choose whether to open or close stations
            if RandomU() < 0.5 // open stations
                numClosed = L/2 - Sum(StepZ)
                toOpen = RandomIntBetween(1, min(5, numClosed))
                candZ = OpenStations(toOpen)
                ConstructSolution()
                activeTrains = Sum(candY)
                for i = 0 to fleetRange - 1
                    fleetSize = activeTrains - Floor(fleetRange/2) + i
                    for j = 0 to 1
                        ConstructFleetSolution(fleetSize, j)
                        if candObj > bestObj - tiThreshold
                            ImproveCandSolution(fleetSize, tCutoff, gCutoff)
                        if candObj > holdObj
                            ReplaceHoldWithCand()
            else // close stations
                numOpen = Sum(StepZ)
                toClose = RandomIntBetween(1, min(5, numOpen - 2))
                candZ = CloseStations(toClose)
                ConstructSolution()
                activeTrains = Sum(candY)
                for i = 0 to fleetRange - 1
                    fleetSize = activeTrains - Floor(fleetRange/2) + i
                    for j = 0 to 1
                        ConstructFleetSolution(fleetSize, j)
                        if candObj > bestObj - tiThreshold
                            ImproveCandSolution(fleetSize, tCutoff, gCutoff)
                        if candObj > holdObj
                            ReplaceHoldWithCand()
```

MAIN CONT'D

```
        stepZ = holdZ
        if holdObj > bestObj
            REPLACEBESTWITHHOLD()
            noImpr = 0
        else
            noImpr+ = 1
            if noImpr > REVISITCUTOFF
                BREAK
    REPLACECANDWITHBEST()
    fleetSize = SUM(candY)
    // try to serve any remaining unserved customers
    POLISHPARTONE()
    // extended version of ImpoveCandSolution()
    IMPROVECANDSOLUTION(fleetSize, POLISHTCUTOFF, POLISHGCUTOFF)
    // shut down trains that aren't even covering own cost, shut down loops that arent covering v
    // attempt to serve customers from these trains elsewhere
    CLEARLIGHTRUNSANDLOOPS()
    if candObj > bestObj
        REPLACEBESTWITHCAND()
```

```
CONSTRUCTSOLUTION()
    for τ = 0 to T − 1
    seedGroup = −1
    // only returns groups that have not served as a seed, if none returns -1
    seedGroup = GETLARGESTUNSERVEDGROUP()
    if seedGroup == −1
        BREAK // all groups tried
    // return w so that train τ can serve seedGroup in the middle of its window
    w_τ = SCHEDULETRAINTOSERVEG(seedGroup)
    // attempt to add all unserved groups to train τ & update x variable
    // train schedule can flex to serve new customers but cannot uncover others to do so
    SERVEALLPOSSIBLEGROUPS(w_tau)
    // call CPLEX to solve MIP - maximize revenue subject to capacity constraint
    // also updates n and q variables based on ridership
    SOLVERHOMIP()
    // check if newly scheduled train is profitable
    if CHECKTRAINPROFITABILITY(τ) == 0
    // remove customers assigned to train τ reset associated variables
        CLEARTRAIN(τ)
        τ− = 1
    else
        candY[τ] = 1
```

```
ConstructFleetSolution(fleetSize, method)
    if method == 0 // largest group
        for τ = 0 to fleetSize − 1
            candY[τ] = 1
            seedGroup = −1
            seedGroup = GetLargestUnservedGroup()
            if seedGroup == −1
                Break
        w_τ = ScheduleTrainToServeG(seedGroup)
        ServeAllPossibleGroups(w_tau)
        SolveRhoMIP()
    else // equidistant starting schedules
            buffer = GetLengthOfLoop()/fleetSize
            for τ = 0 to fleetSize − 1
            w_τ = τ ∗ buffer
            ServeAllPossibleGroups(w_tau)
            SolveRhoMIP()

ImproveCandSolution(fleetSize, tCutoff, gCutoff)
    for i = 0 to impTrainLoops −1
        // choose two trains using RandomIntBetween() with uniqueness check
        // train -1 serves as dummy train for unserved passengers
        train1, train2 = ChooseTrainPair()
        for j = 0 to impGroupLoops − 1
            ReplaceTsihWithCand()
            // choose one group from each train selected above
            group1, group2 = ChooseGroupFromEachTrain(train1, train2)
            temp = RandomU()
            if temp < probGroupSwap// swap two groups
                // serve group on train if possible and update variables
                // set ρ as large as possible for newly served group
                serve1 = ServeGroupOnTrain(group1, train2)
                serve2 = ServeGroupOnTrain(group2, train1)
                if serve1 + serve2 == 2
                    if tsihObj > candObj
                        ReplaceCandWithTsih()
                        gNoImpr = 0
                        tNoImpr = −1
                    else
                        gNoImpr+ = 1
                        if gNoImpr > gCutoff
                            Break
        tNoImpr+ = 1
        if tNoImpr > tCutoff
            Break
```