Analysis of Transportation Network Design Strategies for Forced Transfer Busing

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Abstract

Forced transfer busing occurs primarily at the elementary school level when students are bused to an alternate school when their geographically-assigned school is full at their specific grade level. Ineffective forced transfer busing can result in extra student travel time and inefficient use of often scarce transportation resources. In fact, some force transferred students regularly arrive to their alternate school after morning classes have started due to inefficient transportation practices. We examine various forced transfer busing network design strategies using actual public school system data from two school districts to assess various proposed solution methodologies effectiveness at developing practically implementable busing solutions in a realistic amount of time. In addition, preliminary models and analysis are presented for special needs busing problems in one local school district such that student travel time is minimized for these often medically-fragile children.

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1 Introduction

When a city's growth rate significantly increases in a short period of time, the school district of that city is often faced with difficulties allocating students to schools. Usually, a school will reach maximum capacity before all students in that school's zone have been enrolled. The remaining students are then forced to attend a school in a different school zone. Transferring these students not only requires extra buses and thus extra costs, but also frequently results in thirty to sixty minutes of lost instructional time for the students. Forced transfer students are usually required to leave for school up to two hours before their school start time, wait on their bus and at other schools for long periods of time, and then still arrive late to their own school. Unfortunately, forced transfer students are generally elementary school students. Therefore, these students, who travel and wait for nearly two hours with little supervision, range in age from five to twelve.

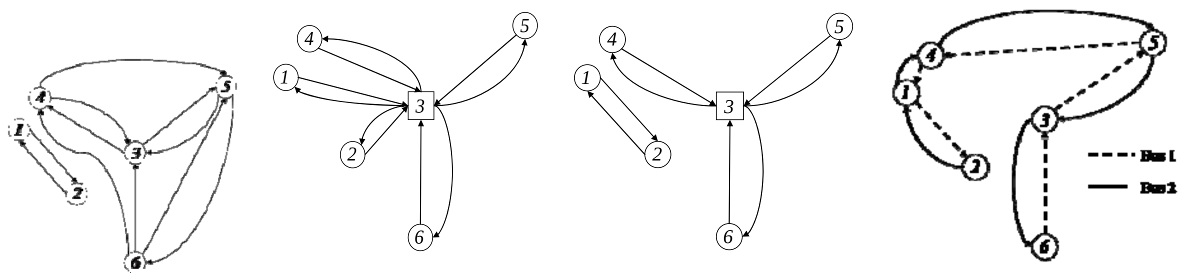
In Springdale, Arkansas there are approximately 170 of these students. The Springdale School District is currently the third largest school district in the state. The district has grown 62% in the last ten years, with most of the growth concentrated on the east side of town. Springdale currently has twenty-two schools, sixteen of which are elementary schools. To accommodate the growth, the city is planning to build one new school per year for the next ten years. In the meantime, the district is struggling to manage school capacity, bus routing and scheduling. Other large school districts in the Northwest Arkansas area use advanced GPS technology to effectively route their buses; however, Springdale lacks the available resources for this technology and resorts to less effective modes of routing. This research is an attempt to find a less expensive solution to the problem of routing forced transfer students in school districts with fewer available means.

In Fort Smith, Arkansas there are currently 185 students being force transferred at the elementary level. The Fort Smith School District is right behind Springdale, as the fourth largest district in Arkansas. The problems Fort Smith must address are similar to those in Springdale. Population growth, along with parental requests for transfer, have caused demand to exceed capacity for many of the nineteen elementary schools. This deficit is overcome by transferring students across town, incurring additional costs and resulting in lost education time. With the currently suffering economy, it is as important as ever to use resources wisely and efficiently, and student classroom time is always a priority. This research attempts to determine the best way to minimize transportation costs without sacrificing classroom time.

2 Previous Research

In conducting this research, previously researched network design strategies were first investigated. Two of these strategies were then used with developed models to analyze real world data. The following describes the network design strategies studied and outlines the two models chosen to analyze the data.

Previous forced transfer research has consisted of four main network design strategies. These strategies are direct pairwise, hub-and-spoke, hybrid and circuit. Examples of each design are shown below in Figure 1. In the direct pairwise strategy, a bus is sent from each school to every alternate school where demand occurs. In the hub-and-spoke strategy, buses are sent from each school to a common location, one of the schools, which is called the hub. Buses then travel from the hub to all other schools as demand occurs. The hybrid strategy is a combination of the direct pairwise and hub-and-spoke strategies. Demand for transfer from a student's home school to an alternate school is met either by direct transfer or through the hub. In circuit routing, buses travel in a loop pattern from school to school. Each school can be visited only once in a circuit. The most basic circuit route that satisfies demand to and from each school has two buses with bus one traveling from school 1 to school 2, 3, 4, etc. and the second bus simultaneously traveling in the reverse order.



Direct Pairwise | Hub and Spoke | Hybrid | Circuit

Figure 1. Network Design Strategies

2.1 Hub-and-Spoke Literature

2.1.1 Pure Hub-and-Spoke Networks

Research concerning hub-and-spoke networks is mainly focused on the hub location problem. The hub location problem involves two decisions: locating hubs and assigning non-hub nodes to hubs. In addition, there typically are three assumptions for the hub location problem:

- The network is complete with a link between every node pair.
- There are economies of scale expressed as a discount factor (α) for using inter-hub connections.
- No direct service (e.g., between two non-hub nodes) is permitted.

Bryan and O'Kelly (1999) classify the hub-and-spoke problem further into two classes: single assignment and multiple assignments. Single assignment means that each node can only be connected to one hub and vice versa, whereas multiple assignments allow each node to be connected to more than one hub. For both types of hub-and-spoke problems, inter-hub connection is also permitted when multiple hubs exist. Bryan and O'Kelly (1999) summarize the hub-and-spoke literature to date in Table 1.

Table 1. Analytical Research on Hub-and-Spoke Networks (Bryan and O'Kelly, 1999)

Single Assignment Model Formulation	Multiple Assignment Model Formulation
O'Kelly (1987)	Campbell (1994b)
Heuristics	*Solution methods*
O'Kelly (1987)	Campbell (1994b)
Klincewicz (1991)	Ernst & Krishnamoorthy (1998)
Abdinnour-Helm & Venkataramanan (1992)	Klincewicz (1996)
Klincewicz (1992)	Skorin-Kapov, Skorin-Kapov,
O'Kelly (1992)	and O'Kelly (1996)
Abdinnour-Helm & Venkataramanan (1993)	
Skorin-Kapov & Skorin-Kapov (1994)	*Exact network based methods*
Aykin (1995)	*for multiple and single allocation models*
O'Kelly, Skorin-Kapov, and Skorin-Kapov (1995)	Ernst & Krishnamoorthy (Forthcoming b)
Campbell (1996)	
Ernst and Krishnamoorthy (1996)	
Smith, Krishnamoorthy, and Palaniswami (1996)	
Linearizations	
Campbell (1994b)	
Ernst & Krishnamoorthy (1996)	
O'Kelly et al. (1996)	
Skorin-Kapov, Skorin-Kapov, and O'Kelly (1996)	
Extensions with Fixed Hub Location	**Extensions with Endogenous Hub Location**
Grove and O'Kelly (1986)	O'Kelly (1986)
Jeng (1987)	Chou (1990)
Flynn and Ratick (1988)	O'Kelly (1992)
Daskin and Panayotopoulos (1989)	Campbell (1993)
Hall (1989)	Aykin (1994)
Miller (1990)	Campbell (1994b)
O'Kelly and Lao (1991)	Aykin (1995)
Dobson and Lederer (1993)	Ernst and Krishnamoorthy (Forthcoming a)
Kuby and Gray (1993)	Jaillet et al. (1996)
	O'Kelly (1998a, 1998b)
	Ebery et al. (1998)
	O'Kelly and Bryan (1998)
Sensitivity Analysis	
O'Kelly et al. (1996)	

O'Kelly (1987) first models the single assignment hub location problem as a quadratic integer program to minimize total network cost under the condition that all inter-hub links are fully interconnected. The network costs are described in three parts: 1) the travel cost from origin to hub, 2) the cost of traveling across the inter-hub link (if necessary), and 3) the travel cost from hub to destination. In O'Kelly (1987), travel costs on inter-hub links are independent of the amount of flow traveling across the link. In particular, although there can be more than one hub in the network, every node is assigned to a single hub. Many heuristics have since been developed to solve this quadratic program:

- Exchange heuristic of Klincewicz (1991)—exchange hubs with non-hub nodes
- Clustering heuristic of Klincewicz (1992)—first cluster nodes into groups, then assign a hub to each group
- Greedy exchange based on maximum flow or minimum transportation cost by Campbell (1996)
- Simulated annealing by Abdinnour-Helm and Venkataramanam (1992)
- Genetic algorithm by Abdinnour-Helm and Venkataramanam (1993)
- Tabu search by Skorin-Kapov and Skorin-Kapov (1994)

To date, the Tabu search of Skorin-Kapov and Skorin-Kapov (1994) has achieved the best solutions.

Rather than analyze the quadratic program directly, another popular approach is to linearize the quadratic model (Campbell (1994), Skorin-Kapov *et al.* (1996), O'Kelly (1996), Ernst and Krishnamoorthy (1996)). Tight linearizations of the quadratic program often can provide integer solutions without enforcing integrality requirements. However, the linearized model can only obtain exact solutions on relatively small problem instances (e.g., 25 nodes).

Irnich (2000) expands the single assignment problem to include multi-depot pickup and delivery, narrow time windows, and heterogeneous vehicles. The task is to find a minimal cost set of trips. Irnich (2000) proposes a two-phase algorithm to solve the problem wherein phase one enumerates all possible route combinations, then phase two assigns transportation requests to the combinations. The method of Irnich (2000) is limited by the number of hub stops on the route—this must be fairly small so that all possible routes can be enumerated easily.

2.1.2 Hybrid Hub-and-Spoke Networks

The hybrid hub-and-spoke network design problem can be classified as either a location-routing problem or a pure routing problem. For the location-routing problem, Aykin (1995) investigates an expansion location-routing problem in which non-stop services are permitted. He builds a nonlinear model to minimize total distance traveled, and then decomposes the model into two subproblems: known hub location and known service type. On the basis of these two subproblems, Aykin (1995) proposes a heuristic that finds hub locations first, and then assigns nodes to hubs while simultaneously determining delivery routings.

The pure routing problem deals with finding optimal routes for known hubs. Liu *et al.* (2003) develop a heuristic procedure based on the Clarke-Wright heuristic (1964) to solve a hybrid hub-and-spoke network with milk runs (i.e., more than one stop can be made during a collection or delivery trip). In their research, Liu *et al.* (2003) assume there is no fixed cost for operating a hub, nor any variable cost incurred when entering or leaving the hub. Further, Liu *et al.* (2003) assume that hub location is known and that there is an infinite supply of vehicles. Liu *et al.* (2003) prove that a hybrid system is better than both pure systems and that the demand distribution is the most important factor in hybrid system design. Finally, Chong *et al*. (2006) develop a heuristic procedure to solve the problem of scheduling and routing shipments in a hybrid system when a set of feasible, discrete inter-shipment times is required. In their heuristic procedure, the hybrid network design is determined via an enumerative strategy.

2.2 Circuit Routing Literature

Another transportation strategy used in practice today to deal with forced transfer busing issues is to use additional, specialized bus routes to transport only forced transfer students. In contrast to regular bus routes which drop off students at their base school, these bus routes travel from school to school within the district, picking up and dropping off only transferred students. In this *circuit routing* strategy, the forced transfer buses travel in a cycle or loop pattern, and each school can be visited at most one time on any circuit. Therefore, students cannot be transferred to an alternate school previously visited on a given bus's route.

Consider a simple circuit bus route with the route $A \rightarrow B \rightarrow C$. This means that the bus first visits School *A*, followed by School *B*, and then School *C*. On this route, students can be transported from *A* to *B*, from *A* to *C*, and from *B* to *C*. However, if a student needs to be transferred from School *C* to School *A*, another bus route would be required, as this is not possible on this current simple route. Therefore, all combinations of origins and destinations could be achieved by having two circuit routes running in opposite directions (i.e., $A \rightarrow B \rightarrow C$ and $C \rightarrow B \rightarrow A$). While this solution would clearly achieve the objective of minimizing the number of bus routes required, as more and more schools are added to the circuit bus routes, additional buses may be required in order to minimize total distance traveled and/or student wait time.

2.2.1 The Vehicle Routing Problem with Pickups and Deliveries and Time Windows

The forced transfer busing problem is similar to the Vehicle Routing Problem with Pickups and Deliveries and Time Windows (VRPPDTW). In the VRPPDTW, all routes are required to begin and end at a common depot, and each vehicle has a capacity that cannot be exceeded. Also, time constraints are added that require service at any particular stop to occur during a specific time interval.

Nagy and Salhi (2006) developed an integrated heuristic to solve the Vehicle Routing Problem with Pickups and Deliveries (VRPPD) in order to find a set of routes which minimize the total distance traveled by the vehicles, subject to maximum distance and capacity constraints. The basic integrated heuristic consists of four phases during which an initial feasible route is constructed and then improved upon using several improvement techniques. In Phase 1, a weakly feasible solution is found using a route construction heuristic. In this weakly feasible solution, both the total pickup and the total delivery of every route are below the maximum capacity constraint. In Phase 2, this initial solution is improved upon while maintaining weak feasibility. In Phase 3, the solution becomes strongly feasible, which refers to a solution in which the load for each individual arc does not exceed the vehicle capacity. Finally, in Phase 4, this solution is improved upon while maintaining strong feasibility.

This basic heuristic can be adapted to solve problems with multiple depots by applying the idea of borderline customers as used in Salhi and Sari (1997). In this sense, a customer is considered borderline when they are located approximately half-way between two depots. An initial solution to the VRPPD is found by first separating the borderline and non-borderline customers. All non-borderline customers are assigned to the nearest depot and weakly feasible solutions are found for each resulting VRPPD. Borderline customers are then inserted into the routes to provide an initial feasible solution, which can be used in Phase 2 of the integrated heuristic.

Several other heuristic solution approaches exist to solve the VRPPDTW. A local search heuristic was presented by Van der Bruggen *et al*. (1993) for the single vehicle problem with minimizing route duration as the objective. This method involves two phases: constructing a feasible route and then iteratively improving upon the solution. The work of Ioachim *et al*. (1995) focused on a clustering algorithm in which customer proximity is used to group customers in order to simplify the routing problem. Lastly, Bent and Hentenryck (2006) introduced a two-stage hybrid algorithm for the VRPPDTW. The first stage aims to minimize the number of routes required using simulated annealing, while the second stages attempts to minimize travel costs using a large neighborhood search.

2.2.2 The Dial-A-Ride Problem

The *Dial-A-Ride (DARP)* problem is similar to the VRPPDTW, except that the requested transport involves persons, rather than goods. This is a typical problem which applies to the transportation of the elderly or disabled in urban areas. In the DARP, users make requests for transportation from a specific origin to a specific destination, and transportation is carried out by vehicles based at a common depot. Also, time windows are pre-specified which bound the arrival and/or departure time of the users. The DARP is NP-hard because it generalizes both the VRPPD and the Traveling Salesman Problem with Time Windows (TSPTW) (Cordeau 2006).

DARP problems are unique in that operating costs and user inconvenience often are weighted against each other when designing a solution. When referring to operating costs, fleet size and travel distance are minimized. When referring to user inconvenience, deviations from desired pickup and drop-off times and excess ride time are minimized. Typically, these competing objectives are balanced by minimizing operating costs subject to service quality constraints. Therefore, the overall goal of the DARP is to design a set of least cost vehicle routes which satisfy capacity, duration, time window, and ride-time constraints.

Several route construction and improvement heuristics have been developed for the DARP. Jaw *et al*. (1986) develop an insertion heuristic which takes time windows directly into account by balancing the preferences of the users with the costs of operation. In this heuristic algorithm, transfer requests are selected and inserted into the vehicle schedule in order of increasing earliest pickup times. Cordeau and Laporte (2005) proposed a tabu search metaheuristic for the DARP. Their algorithm begins with an initial feasible solution, and then moves to the best solution within the neighborhood of the current one. Neighborhood evaluation is based on minimum route duration and minimum ride times. Cycling back to previously visited solutions is avoided by preventing the algorithm from proceeding to a solution that is considered tabu.

Cordeau (2006) introduces a mixed-integer programming formulation to find a set of routes that minimize total routing cost while satisfying capacity and service constraints. Several valid inequalities which strengthen the LP-relaxation of the model are also described for use in a branch-and-cut algorithm. Before the branch-and-cut algorithm is applied, several preprocessing techniques are performed to reduce problem instance size. Next, the algorithm first solves the LP-relaxation. If the relaxation solution is integer, then the optimal solution has been identified; otherwise, an enumeration tree is developed, and separation heuristics are used at each node in the tree to identify violated valid inequalities. If one or more violated inequalities are identified, the cuts are applied and the relaxation model is solved again. If no violated inequalities can be found at a particular node, then the algorithm stops processing at that node. This branch-and-cut algorithm is not suitable for large-scale problem instances; however, on small to medium-size instances, the algorithm reduces both computation time as well as the size of the branch-and-bound tree.

3 Mathematical Models for Forced Transfer Busing

3.1 Hybrid Hub-and-Spoke Model

Based on our literature review, it is clear that hybrid hub-and-spoke networks are at least as good as (and often better than) pure hub-and-spoke and direct busing network designs, as the hybrid strategy accommodates both of these approaches in a single methodology. Therefore, we now develop a mathematical program for hybrid hub-and-spoke forced transfer busing that allows for both hub-and-spoke and direct bus transportation between schools (nodes). In this initial model, we assume that buses do not make any intermediate stops between leaving their origin and arriving at their intended destination (i.e., the hub or another school when performing a direct transport).

For both objective functions, we seek to minimize the performance measure. Therefore, two independent optimization models, each with its own objective function, have been developed. In both model cases, the following assumptions are made:

- Each bus has a single, primary pick-up and drop-off location.
- There exists a finite number of buses available for routing, each of which has an infinite capacity for students.
- Demand between each pair of schools cannot be split across multiple transportation resources (e.g., all demand from location *i* to location *j* must be accommodated on the same bus).

3.1.1 Notation

<u>Sets and Parameters</u>

N	Set of all nodes, $N = \{1,..., n\}$
$d_{i,j}$	Distance (in miles) from node *i* to node *j* ($i \in N$, $j \in N$)
$t_{i,j}$	Bus travel time from node *i* to node *j* ($i \in N$, $j \in N$)
$Q_{i,j}$	=1 if there exists student forced transfer demand from node *i* to node *j*; otherwise, =0 ($i \in N, j \in N, Q_{i,i} = 0$)

b_i	Time a bus leaves school i ($i \in N$)
s	Start time for all schools
r	Number of buses available
n_d	Number of demand pairs ($\sum_i \sum_{j \neq i} Q_{i,j}$)

Decision Variables

$x_{i,j}$	=1 if students are transferred from node i to node j directly; otherwise, =0 ($i \in N, j \in N, x_{i,i} = 0$)
$y_{i,j}$	=1 if students are transferred from node i to node j through the hub; otherwise, =0 ($i \in N, j \in N, y_{i,i} = 0$)
z_i	=1 if node i is chosen as the hub node; otherwise, =0 ($i \in N$)
$o_{i,j}$	=1 if a bus is used to transfer students from node i to hub j; otherwise, =0 ($i \in N, j \in N, i \neq j$)
$e_{i,j}$	=1 if a bus is used to transport students from hub i to node j; otherwise, =0 ($i \in N, j \in N, i \neq j$)
v	Total transportation miles, ≥ 0
w_j	Hub departure time of bus bound for school j ($j \in N$) under the leave-when-ready hub policy, ≥ 0
δ	Time buses leave the hub under the leave-simultaneously hub policy, ≥ 0
f	Number of buses used at the hub, ≥ 0
h	Number of buses used for direct transport, ≥ 0
L	Maximum lateness of all the buses, ≥ 0
u_i	Arrival time of bus travelling from node i to the hub ($i \in N$), ≥ 0
$k_{i,j}$	Arrival time of bus transporting demand pair (i, j) directly ($i \in N, j \in N$), ≥ 0
$g_{i,j}$	Arrival time of bus transporting demand pair (i, j) through the hub ($i \in N, j \in N$), ≥ 0
$a_{i,j}$	Number of minutes bus travelling from node i to node j is late ($i \in N, j \in N$), ≥ 0

3.1.2 Mixed-Integer Program

The objective is to minimize total transportation miles as shown in objective (1) or minimize maximum bus lateness (#2). The secondary objective terms in each objective function are scaled by 0.001 as a means of breaking ties when alternative optimal solutions exist.

$$\min \; v + 0.001L \tag{1}$$

$$\min \ L + 0.001v \tag{2}$$

Constraint set (3) guarantees exactly one and only one hub exist.

$$\sum_i z_i = 1 \tag{3}$$

In addition, a lower bound on the number of hubs is given by constraint set (4).

$$\sum_i z_i \geq \left(\sum_i \sum_j y_{i,j}\right) \Big/ n_d \tag{4}$$

Constraint set (5) assures student forced transfer demand must be satisfied either via direct or indirect (i.e., hub-and-spoke) bus transportation.

$$y_{i,j} + x_{i,j} = Q_{i,j} \qquad i \in N, j \in N \tag{5}$$

Upper and lower bounds for decision variable $o_{i,j}$ is computed by constraint sets (6) and (7).

$$o_{i,j} \leq \sum_l y_{i,l} \qquad i \in N, j \in N \tag{6}$$

$$e_{i,j} \leq \sum_l y_{l,j} \qquad i \in N, j \in N \tag{7}$$

Upper and lower bounds for decision variable $e_{i,j}$ is computed by constraint sets (8) and (9).

$$o_{i,j} \geq \left(\sum_l y_{i,l} \Big/ n \right) + z_j - 1 \qquad i \in N, j \in N \tag{8}$$

$$e_{i,j} \geq \left(\sum_l y_{i,l} \Big/ n \right) + z_i - 1 \qquad i \in N, j \in N \tag{9}$$

Next, constraint set (10) allows that at most one bus can leave from each node *i*, while constraint set (11) permits at most one bus to arrive at each node *j*.

$$\sum_j o_{i,j} \leq 1 \qquad i \in N \tag{10}$$

$$\sum_{i} e_{i,j} \leq 1 \qquad j \in N \tag{11}$$

On the basis of the values of the decision variables $o_{i,j}$ and $e_{i,j}$, the total number of buses used in hub-and-spoke transportation is calculated by constraint sets (12) and (13).

$$f \geq \sum_{i} \sum_{j} o_{i,j} \tag{12}$$

$$f \geq \sum_{i} \sum_{j} e_{i,j} \tag{13}$$

Similarly, constraint set (14) calculates the number of buses used for direct transportation.

$$h = \sum_{i} \sum_{j} x_{i,j} \tag{14}$$

Constraint set (15) guarantees that the total number of buses used does not exceed the number of buses available.

$$f + h \leq r \tag{15}$$

Constraint sets (16) and (17) calculate the arrival time of the bus travelling from origin location i to the hub. Specifically, constraint set (16) focuses on the demand pairs originating at schools rather than the hub, while constraint set (17) pertains to hub-originating demand pairs.

$$u_i \geq \sum_{i \neq l} o_{i,l} \left(b_i + t_{i,l} \right) \qquad i \in N \tag{16}$$

$$u_i \geq b_i z_i \qquad i \in N \tag{17}$$

The hub departure time of the bus headed to school j is calculated by constraint set (18) when the leave-when-ready hub policy is in effect. In (18), the value of M ("big M") is set as the largest possible value of w_j, which is the sum of the maximum school ready time and the longest amount of time spent on any bus route from a school to the hub ($M_1 = \max_{i \in N} \left(b_i \right) + \max_{i \in N, j \in N, i \neq j} \left(t_{i,j} \right)$).

$$w_j \geq u_i - \left(1 - y_{i,j} \right) M_1 \qquad i \in N, j \in N \tag{18}$$

On the basis of constraint set (18), the leave simultaneously time is obtained in (19).

$$\delta \geq w_j \qquad j \in N \tag{19}$$

Constraint sets (20) and either (21) or (22) calculate upper and lower bounds on each bus's arrival time to its destination for demand pairs transported through the hub, depending on the hub policy being used. Under the leave simultaneously policy, the lower bound is computed by (21), while the lower bound for the leave-when-ready policy is computed using (22). In constraint sets (20), (21), and (19-lwr) the value of M is set to the biggest possible bus arrival time, which is the sum of the largest school ready time and the longest time spent on a bus route ($M_2 = \max_{i \in N} b_i + 2\left[\max_{i \in N, j \in N, i \neq j}\left(t_{i,j}\right)\right]$).

$$g_{i,j} \leq My_{i,j} \qquad i \in N, j \in N \tag{20}$$

$$g_{i,j} \geq \delta + \sum_{l \neq j} t_{l,j} e_{l,l} - M_2(1 - y_{i,j}) \qquad i \in N, j \in N \tag{21}$$

$$g_{i,j} \geq w_j + \sum_{l \neq j} t_{l,j} e_{l,l} - M_2(1 - y_{i,j}) \qquad i \in N, j \in N \tag{22}$$

Finally, considering the two individual objective functions of interest, additional computations are required to properly compute our time-based metric of maximum bus lateness time in (23)-(25).

$$k_{i,j} = \left(b_i + t_{i,j}\right)x_{i,j} \qquad i \in N, j \in N \tag{23}$$

$$a_{i,j} = g_{i,j} + k_{i,j} - s \qquad i \in N, j \in N \tag{24}$$

$$L \geq a_{i,j} \qquad i \in N, j \in N \tag{25}$$

3.2 Circuit Routing Model

Based on our literature review, we are motivated to assess the efficacy of circuit routing strategies for the forced transfer busing problem. Therefore, we now develop a mathematical program for the circuit routing case.

3.2.1 Notation

Sets and Parameters

I:	Set of nodes ($I = \{1,...,n\}$)
R:	set of routes ($R = \{1,...,a\}$)
n	total number of bus stops, *a* is the number of buses available.
$h_{i,j}$	Distance between node *i* and node $j, i \in I, j \in I$
$Q_{i,j}$	1, if there is at least one student transferred from *i* to *j*, otherwise 0, $i \in I, j \in I$
C_i	Candidates of origin point, $i \in I$

Decision Variables

$x_{r,i,j}$	1, If node *i* is followed by *j* on route *r*, otherwise 0, $i \in I, j \in I, (i \neq j), r \in R$
y_r	1, if route r is used, otherwise 0, $r \in R$
$z_{i,r}$	1, if node *i* is the origin point on route r, otherwise 0, $i \in I, r \in R$
$o_{i,r}$	1, if node *i* is the destination point on route r, otherwise 0, $i \in I, r \in R$
$d_{i,r}$	1, if node *i* is on route r, otherwise 0, $i \in I, r \in R$
t_r	total distance/time for each route $r \in R$
$U_{r,i,j}$	1 if there is at least one student need to be transferred from *i* to *j* on the route *r*, otherwise 0, $i \in I, j \in I, (i \neq j), r \in R$
$p_{i,r}$	positions of node i on the route r, the big number means the former position, $i \in I, r \in R$

3.2.2 Mixed-Integer Program

The objective is to minimize total bus distances:

$$\text{Minimize } \sum_r t_r \tag{26}$$

First, citing constraint set (27), origin points must be selected from candidate point. This constraint is used to reduce the range of origin points.

$$\sum_{r} o_{i,r} \leq C_i \qquad i \in I \tag{27}$$

Constraint sets (28) and (29) guarantee that the total number of origins/destinations is equal to the number of routes.

$$\sum_{i} o_{i,r} = y_r \qquad r \in R \tag{28}$$

$$\sum_{i} d_{i,r} = y_r \qquad r \in R \tag{29}$$

In addition, constraint sets (30) and (31) confirm that the origin and destination of each route is on the route.

$$o_{i,r} \leq z_{i,r} \qquad i \in I, r \in R \tag{30}$$

$$d_{i,r} \leq z_{i,r} \qquad i \in I, r \in R \tag{31}$$

The total number of origins for all the used routes is equal to the total number of destinations for all the used routes determined in constraint set (32).

$$\sum_{r}\sum_{i} o_{i,r} = \sum_{r} y_r \tag{32}$$

Constraint set (33) certifies that the total number of origins for all the routes is equal to the total number of routes used.

$$\sum_{r}\sum_{i} o_{i,r} = \sum_{r}\sum_{i} d_{i,r} \tag{33}$$

The total number of original points is at least equal to 1 as demonstrated in constraint set (34).

$$\sum_{r}\sum_{i} o_{i,r} \geq 1 \tag{34}$$

Constraint set (35) guarantees that all the nodes must be assigned on at least one route.

$$z_{i,r} \geq 1 \qquad i \in I, r \in R \tag{35}$$

Constraint set (36) is used to decide the logic relations between x and the other decision variables. The basis is that there is sequence between *i* and *j* only when both *i* and *j* are on the route *r*.

$$x_{r,i,j} \leq \left(z_{i,r} + z_{j,r}\right)/2 \qquad i \in I, j \in I, (i \neq j), r \in R \tag{36}$$

At the same time, there is no sequence between *i* and *j* on route *r* if route r is not used as expressed by constraint set (37).

$$x_{r,i,j} \leq y_r \qquad i \in I, j \in I, (i \neq j), r \in R \tag{37}$$

Constraint sets (38) and (39) assure the original points must be followed by other nodes and the destination point must have nodes in front of it.

$$\sum_j x_{r,i,j} \geq o_{i,r} \qquad i \in I, r \in R \tag{38}$$

$$\sum_i x_{r,i,j} \geq d_{j,r} \qquad j \in I, r \in R \tag{39}$$

Constraint sets (40) and (41) make sure that only one node can be connected with the other node on one route.

$$\sum_j x_{r,i,j} \leq 1 \qquad i \in I, r \in R \tag{40}$$

$$\sum_i x_{r,i,j} \leq 1 \qquad j \in I, r \in R \tag{41}$$

Further, one and only one node can be connected to the other (former or latter) on the condition that the node is on the route:

$$\sum_i x_{r,i,j} + o_{j,r} = z_{j,r} \qquad j \in I, r \in R \tag{42}$$

$$\sum_j x_{r,i,j} + d_{i,r} = z_{i,r} \qquad i \in I, r \in R \tag{43}$$

Constraint set (44) requires that each connection is directed and has only one direction:

$$x_{r,i,j} + x_{r,j,i} \leq 1 \qquad i \in I, j \in I, (i \neq j), r \in R \tag{44}$$

Further, an arc can't be used by more than one route.

$$\sum_r x_{r,i,j} \leq 1 \qquad i \in I, j \in I, (i \neq j) \tag{45}$$

Constraint set (46) is used to calculate the total time/distance spent on each route:

$$t_r = \sum_i \sum_i x_{r,i,j} * h_{i,j} \qquad r \in R \tag{46}$$

In addition to the other constraint described above for the circuit routing method, we need additional constraint sets to model the continuity of student travel. The position of nodes on each route is resolved by constraint set (47):

$$p_{i,r} \geq p_{j,r} + 1 - (1 - x_{r,i,j}) * M \qquad i \in I, j \in I, (i \neq j), r \in R \tag{47}$$

In addition, the position number of destination node is set to 1 in constraint set (48).

$$p_{i,r} \geq d_{i,r} \qquad i \in I, r \in R \tag{48}$$

The $U_{r,i,j}$ values are decided by position number with the following constraint set:

$$U_{r,i,j} \leq (U_{i,r} - U_{j,r}) / M + (z_{i,r} + z_{j,r}) / 2 \qquad i \in I, j \in I, (i \neq j), r \in R \tag{49}$$

Furthermore, if at least one student needs to be transferred from i to j on route *r*, then node *i* and node *j* must both assigned on the route *r*:

$$U_{r,i,j} \leq (z_{i,r} + z_{j,r}) / 2 \qquad i \in I, j \in I, (i \neq j), r \in R \tag{50}$$

The final constraint sets makes sure to meet all the student transfer requirements.

$$\sum_r U_{r,i,j} \geq Q_{i,j} \qquad i \in I, j \in I, (i \neq j) \tag{51}$$

4 Forced Transfer Busing Case Studies

Forced transfer problems occur primarily at the elementary level, so this research uses data from the elementary schools in both the Fort Smith and Springdale school districts. The demand data provided by both school districts with the corresponding distance and time data was first implemented in the hybrid hub-and-spoke model developed by Jia (2008) to determine the best hub location, the total bus miles, the maximum time a student will arrive late to school, the average lateness, the number of buses used and the number of direct pair transfers. The data was analyzed with three different methodologies. In the first method, each bus was allowed to leave when ready, or when all students had boarded that particular bus and the objective was to minimize total bus miles ("Dist" objective). The data was then analyzed with the buses still allowed to leave when ready, but with the objective to minimize maximum lateness ("Late-LWR" objective). Finally, the data was run with the buses required to leave at the same time and with the objective set to minimize the maximum late time a student arrives ("Late-DS" objective). The bus start time was simulated throughout the experimentation. The demand and distance data was then used in the circuit routing model above to determine the best circuit route combinations, the number of buses needed, and total miles. The results from testing these models with data from both Fort Smith and Springdale are detailed in the following sections.

4.1 Fort Smith Public Schools

The Fort Smith School District currently has nineteen elementary schools, located as shown in Figure 2. The current bus routes are shown in Figure 3. Four buses run different circuit routes to meet all elementary-level demand. The demand data to be analyzed is shown below in Table 2.

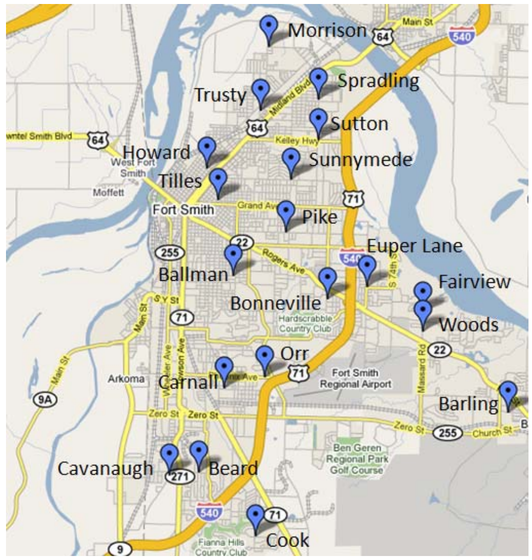


Figure 2 Fort Smith Elementary Schools (50.3 mi^2)

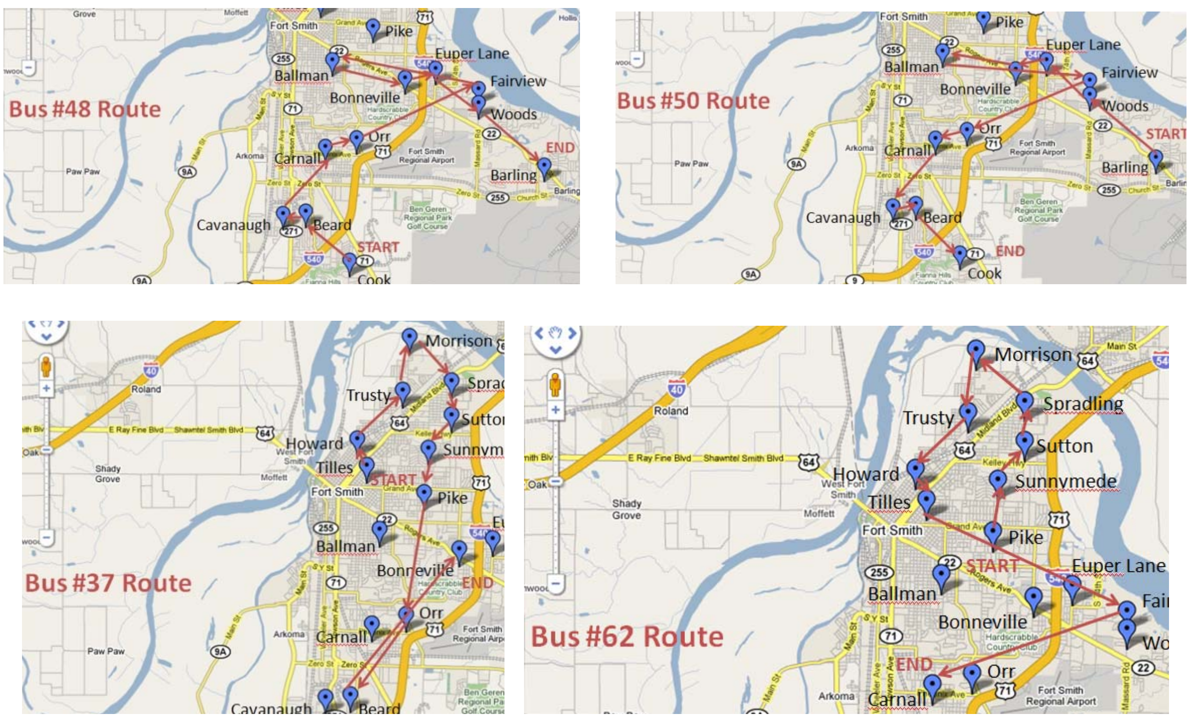


Figure 3: Current Ft. Smith Routes

Table 2: Fort Smith Elementary Demand Data

	Ballm	Barling	Beard	Bonnevil	Carnall	Cavar	Cook	Euper	Fairview	Howard	Morris	Orr	Pike	Spradling	Sunnymed	Sutton	Tilles	Trusty	Woods
Ballman	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
Barling	0	0	1	0	0	0	0	1	0	0	0	5	0	0	0	0	0	0	4
Beard	0	0	0	1	0	0	5	0	0	0	0	1	0	0	0	0	0	0	0
Bonneville	0	0	1	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	5
Carnall	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Cavanaugh	0	0	4	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Cook	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Euper	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	4
Fairview	7	0	3	2	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0
Howard	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0
Morrison	0	0	0	0	3	0	0	0	0	4	0	0	0	1	1	0	3	0	0
Orr	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Pike	1	0	1	0	1	0	2	0	0	1	0	4	0	7	0	0	0	2	0
Spradling	0	0	0	0	0	0	0	0	0	1	0	1	0	0	2	0	1	6	0
Sunnymede	0	0	2	0	0	0	0	0	0	0	0	3	3	0	0	7	0	2	0
Sutton	0	0	0	0	0	0	4	0	0	0	0	2	1	5	1	0	2	1	0
Tilles	5	0	4	0	1	0	0	0	0	4	0	0	0	1	0	0	0	1	0
Trusty	0	0	0	0	0	0	0	0	0	0	0	2	1	3	0	3	8	0	0
Woods	0	0	0	0	2	0	0	1	0	0	0	2	0	0	0	0	0	0	0

Table 2 shows demand from the schools in the left column to the schools along the top row. For example, the demand from Fairview to Beard is three (i.e., 3 students must be transferred from Fairview to Beard).

4.1.1 FSPS Hybrid Hub-and-Spoke Model

We have used the hybrid hub-and-spoke model to analyze an alternative busing strategy for Fort Smith. We hope to offer district decision makers choices in their approach to forced transfer busing, and hopefully suggest alternatives that will save transportation costs and student ride and wait time. No data was provided from Fort Smith describing exactly when a transfer bus is ready to leave a particular school (after all regular bus routes have arrived). We created five different data sets to be analyzed in the model based on our estimates for these times.

All data sets are based on a school start time of 8:00 a.m. The first data set assumes all buses can leave as early as 7:30 and the second assumes all buses are ready to leave at 8:00. These give us a frame of reference: one assumes an unusually early leave time and the other, a very late leave time. In the remaining three data sets, times within 5 minutes of 7:45 are randomly assigned to each school. Also note that we did not receive any data telling us which schools could feasibly be the hub. Thus we assumed all schools were eligible. Tables 3 through 7 show the model results using each of the five described data sets. We obtain the suggested hub, total bus miles and the maximum lateness of any bus used to transport students. We changed the number of available buses we allowed the model until we obtained feasible results. As we changed the number of buses from this feasible number (17), results either became infeasible or did not change. Therefore, only the model results using 17 buses are shown below.

Table 3. Buses leave at 7:30

Objective	# Buses	Hub	Total miles	# direct	# hub	Max Lateness
Dist	17	Ballman	106.16	2	15	0
Late-LWR	17	Ballman	106.16	2	15	0
Late-DS	17	Ballman	106.16	2	15	0

Table 4. Buses leave at 8:00

Objective	# Buses	Hub	Total miles	# direct	# hub	Max Lateness
Dist	17	Ballman	106.16	2	15	30
Late-LWR	17	Euper Lane	135.47	2	15	26
Late-DS	17	Euper Lane	135.47	2	15	26

Table 5. Buses leave around 7:45a

Objective	# Buses	Hub	Total miles	# direct	# hub	Max Lateness
Dist	17	Ballman	106.16	2	15	15
Late-LWR	17	Orr	150.04	2	15	11
Late-DS	17	Euper Lane	135.47	2	15	12

Table 6. Buses leave around 7:45b

Objective	# Buses	Hub	Total miles	# direct	# hub	Max Lateness
Dist	17	Pike	120.58	2	15	13
Late-LWR	17	Sutton	151.29	2	15	12
Late-DS	17	Sutton	152.26	2	15	12

Table 7. Buses leave around 7:45c

Objective	# Buses	Hub	Total miles	# direct	# hub	Max Lateness
Dist	17	Ballman	106.16	2	15	14
Late-LWR	17	Euper Lane	135.47	2	15	10
Late-DS	17	Euper Lane	135.47	2	15	11

Taking a closer look at Table 3, we see that for all three objectives, the suggested hub is Ballman, there are 2 buses used for direct transportation and 15 for demand through the hub, the maximum student lateness is 0 (all arrive on time), and the total mileage is 106.16 miles. Similar observations can be made by examining each of the remaining tables. Note that in other scenarios the data varies with the different objectives. For example, for the 8:00 ready-time case, if the objective is to minimize distance, Ballman is the suggested hub. However, if the objective is to minimize student lateness, Euper Lane is the suggested hub. Notice that for the 7:30 ready-time case, no bus arrives late. This is because this case assumes all regular bus routes arrive to the home schools at least 30 minutes before the school day begins. When the ready-time is increased to 8:00, maximum lateness increases as much as 30 minutes because by the time these buses are ready to leave, the students are already late. The suggested Fort Smith hubs are shown below in Figure 4.

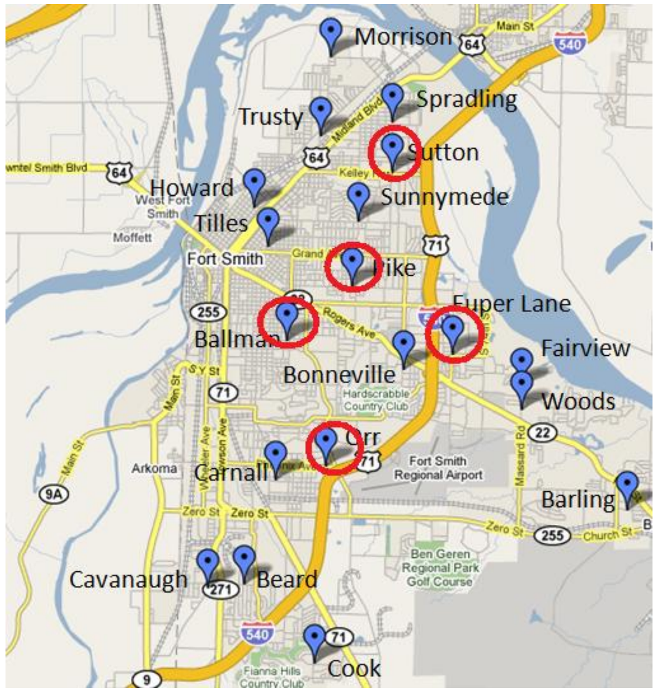


Figure 4: Fort Smith Suggested Hubs

The next step in this research is to meet with Fort Smith officials to compare model results with actual results. When the model performs like the current system, it is validated. If it does not, the model will be adjusted so that better results can be obtained. We also plan to learn which schools are feasible to be the hub. There are a variety of reasons a school may not be allowed to be the hub. For example, there may be limited parking space, not enough room for a bus to turn around, etc. After talking to FSPS officials, we will limit the schools allowed to be the hub in our model and reanalyze the data. We can then compare our new results with the initial results to see how much the hub limitation affects our objectives.

Part of this analysis will include changing the number of buses we provide to the model to see if any new solutions result. We will also compare the number of buses needed to make the model feasible to the current number used by Fort Smith. We will also compare the model results with current practices to look for ways to improve Fort Smith's busing strategy in terms of cost and student ride time. We hope to offer them alternate solutions so that decision makers can be well-informed and allocate their resources wisely.

4.1.2 FSPS Circuit Model

Fort Smith currently uses a circuit strategy for forced transfer busing. The four current elementary-level routes are shown above in Figure 3. In a circuit strategy, a bus visits a series of schools and students can be picked up or dropped off at any school visited along the route. We developed a circuit routing model to analyze Fort Smith's data. Our model is capable of analyzing problems instances over a variety of number of bus cases. We analyze this problem for the smallest feasible number of buses case, two, and are able to produce the optimal solution as given in Tables 8 and 9.

Table 8. FSPS Two Bus Optimal Circuit Solution—First Bus

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
1	Barling	0	0	11	0	11
2	Woods	2.63	4	5	4	12
3	Euper	4.53	8	2	2	12
4	Bonneville	5.97	12	5	0	17
5	Pike	7.86	16	19	0	36
6	Sunnymede	8.95	18	14	0	50
7	Sutton	10.21	20	14	7	57
8	Spradling	11.01	21	9	12	54
9	Morrison	12.76	25	10	0	64
10	Trusty	14.1	29	10	11	63
11	Howard	15.98	34	9	6	66
12	Tilles	16.85	37	10	23	53
13	Ballman	18.8	44	0	6	47
14	Fairview	20.18	48	9	0	56
15	Orr	22.56	53	0	31	25
16	Carnall	23.78	56	0	7	18
17	Cavanaugh	26.56	63	5	0	23
18	Beard	27.13	64	5	16	12
19	Cook	29.18	70	0	12	0

Table 9. FSPS Two Bus Optimal Circuit Solution—Second Bus

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
1	Beard	0	0	2	0	2
2	Cavanaugh	0.57	1	2	0	4
3	Carnall	3.35	8	1	2	3
4	Orr	4.57	11	2	2	3
5	Fairview	6.95	16	9	1	11
6	Ballman	8.33	20	1	7	5
7	Tilles	10.28	27	6	0	11
8	Howard	11.15	30	0	4	7
9	Trusty	13.03	35	7	1	13
10	Morrison	14.37	39	2	0	15
11	Spradling	16.12	43	2	5	12
12	Sutton	16.92	44	2	3	11
13	Sunnymede	18.18	46	3	4	10
14	Pike	19.27	48	0	5	5
15	Bonneville	21.16	52	5	4	6
16	Euper	22.6	56	4	1	9
17	Woods	24.5	60	0	9	0

Examining the results in Table 8 reveals that an exceedingly large number of students are required to be on the circuit bus at the same time (a maximum of 66). In addition, the 70 minutes of required travel time are excessive in that students may be forced to arrive at their destination school late or early stop bus riders will be asked to arrive at their embarkation point very early in the morning. Conversations with FSPS personnel confirmed these shortcomings of the two bus optimal circuit routing solutions. With this in mind, we analyzed the FSPS problem using three circuit buses. The optimal results from this analysis are given in Tables 10, 11, and 12 for each of the resulting circuit bus routes.

Table 10. FSPS Three Bus Optimal Circuit Solution—First Bus

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
1	Tilles	0	0	6	0	6
2	Howard	0.87	3	0	4	2
3	Trusty	2.75	8	7	2	8
4	Morrison	4.09	12	2	0	10
5	Spradling	5.84	16	2	5	7
6	Sutton	6.64	17	2	3	6
7	Sunnymede	7.9	19	3	4	5
8	Pike	8.99	21	0	5	0

Table 11. FSPS Three Bus Optimal Circuit Solution—Second Bus

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
1	Barling	0	0	11	0	11
2	Woods	2.63	4	5	4	12
3	Euper	4.53	8	2	2	12
4	Bonneville	5.97	12	5	0	17
5	Pike	7.86	16	19	0	36
6	Sunnymede	8.95	18	14	0	50
7	Sutton	10.21	20	14	7	57
8	Spradling	11.01	21	9	12	54
9	Morrison	12.76	25	10	0	64
10	Trusty	14.1	29	10	11	63
11	Howard	15.98	34	9	6	66
12	Tilles	16.85	37	10	23	53
13	Ballman	18.8	44	0	6	47
14	Fairview	20.18	48	9	0	56
15	Orr	22.56	53	0	31	25
16	Carnall	23.78	56	0	7	18
17	Cavanaugh	26.56	63	5	0	23
18	Beard	27.13	64	5	16	12
19	Cook	29.18	70	0	12	0

Table 12. FSPS Three Bus Optimal Circuit Solution—Third Bus

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
1	Beard	0	0	2	0	2
2	Cavanaugh	0.57	1	2	0	4
3	Carnall	3.35	8	1	2	3
4	Orr	4.57	11	2	2	3
5	Fairview	6.95	16	9	1	11
6	Ballman	8.33	20	1	7	5
7	Bonneville	10.87	27	5	4	6
8	Euper	12.31	31	4	1	9
9	Woods	14.21	35	0	9	0

Observing the results in Table 11, we still see a large number of students present on the bus in the middle of its route. This optimal result combines with the Table 10 and Table 12 bus recommendations, which are far less crowded and much shorter routes. In fact, when we run the optimization model to add a fourth possible bus, we obtain the same solution as this three bus case. Pike, Sunnymede, and Sutton schools have significant numbers of student that board at each school, leading to this large number of students on the bus. In the future, we need to add additional model constraints to attempt to minimize and/or balance the number of students on each circuit transfer bus route, as well as to maintain some maximum acceptable length of bus riding time for the students.

4.2 Springdale Public Schools

The Springdale Public School District currently has sixteen elementary schools, which are divided into eight east and eight west schools by Highway 71 Business. The location of these schools is depicted below in Figure 5.

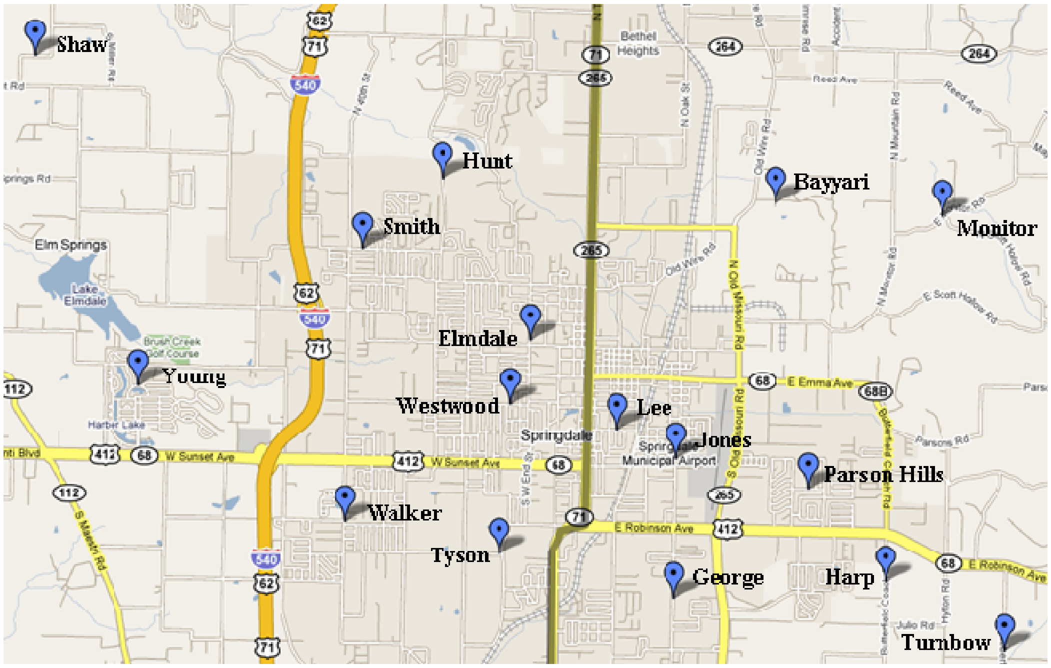


Figure 5. Springdale Elementary Schools

Springdale's current policy does not allow any forced transfers to occur across Highway 71 Business; therefore, they presently use two hub-and-spoke networks: one for the eight east schools and one for the eight west schools. Consequently, we performed our analysis with both models by separating the data into east and west regions. The east hub is currently Harp Elementary and the west hub is Smith Elementary. These schools were chosen by Springdale because they are centrally located, allow for ease of traffic and have adequate space for bus parking. In each instance bus mileage increased as a result of limiting the hubs, but there was not a hugely significant increase in lateness. As illustrated in Figure 6, the current Springdale east side policy assigns Harp Elementary as the hub with seven buses traveling to the other seven schools with no direct pairs. The current west route is displayed in Figure 7 (where the black arrows represent the first set of buses traveling to the hub and the blue arrows represent the buses leaving the hub).

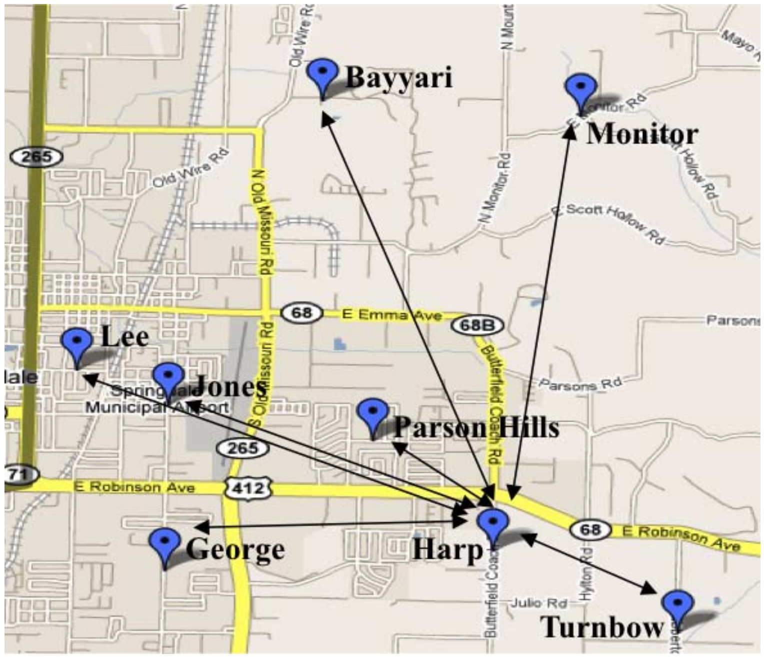


Figure 6: Springdale East Side Current Route

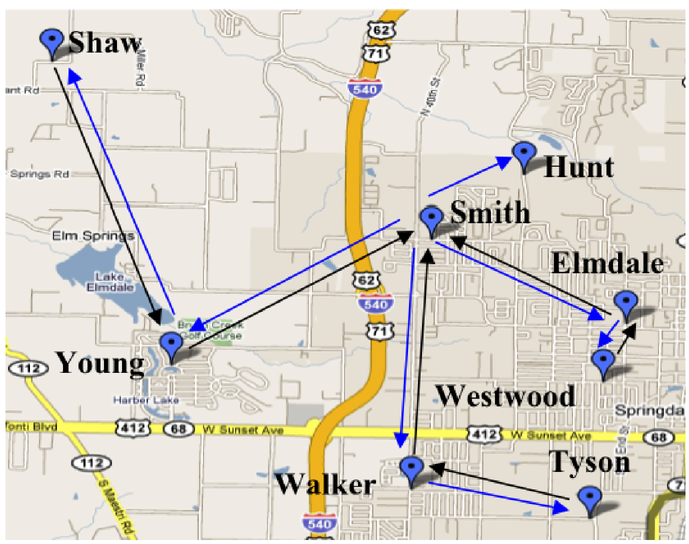


Figure 7: Springdale West Side Current Route

For each set of eight schools (east and west), data on the distance and time between each school was collected as well as the transfer demand between each school and the number of students transferred. The demand data is displayed below in Table 13 and Table 14.

Table 13. Springdale East Schools Transfer Demand

From/To	Bayyari	George	Harp	Jones	Lee	Monitor	Parson Hills	Turnbow
Bayyari	0	0	3	5	8	4	1	4
George	0	0	0	1	2	7	7	1
Harp	0	2	0	0	13	3	3	0
Jones	0	1	1	0	3	0	4	0
Lee	1	0	0	0	0	2	0	1
Monitor	3	0	0	0	0	0	15	2
Parson Hills	0	0	5	3	0	0	0	0
Turnbow	8	0	1	12	0	3	3	0

Table 14. Springdale West Schools Transfer Demand

From/To	Elmdale	Hunt	Shaw	Smith	Tyson	Walker	Westwood	Young
Elmdale	0	0	0	0	0	0	1	0
Hunt	0	0	0	0	0	0	0	0
Shaw	0	5	0	1	0	4	0	0
Smith	3	0	0	0	0	0	0	6
Tyson	2	0	0	4	0	3	0	2
Walker	0	0	0	0	0	0	0	0
Westwood	0	0	0	0	0	0	0	4
Young	0	0	0	0	0	0	0	0

4.2.1 SPS Hybrid Hub-and-Spoke Model

This research first used the Springdale data in the hybrid hub-and-spoke model to test Springdale's current policy and attempt to find an improved solution. Springdale provided a list of schools that were deemed infeasible hub locations due to location or bus parking capacity. With each of the three methods of testing the data, the data was run with all hubs possible and with the hub limited for Springdale. The east results for minimizing total bus miles ("Dist" objective) are given in Table 15. For the minimizing maximum bus lateness objective, Table 16 (Table 17) displays the results for the east schools under the Late-LWR (Late-DS) objective.

Table 15. East Schools: Leave When Ready, Objective: Minimize Miles

All Hubs Possible

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
7	11	-2.3	34.8	Jones	0

Hubs limited to Bayyari, Harp, and Turnbow

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
7	13	-1.3	41.9	Harp	0

Table 16. East Schools: Leave When Ready, Objective: Minimize Maximum Late

All Hubs Possible

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
10	5	-2.6	43.4	Jones	3
7	11	0.6	34.8	Jones	0

Hubs limited to Bayyari, Harp, and Turnbow

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
11	5	-2.5	53.6	Harp	4
8	7	-1.3	45.1	Harp	1
7	13	1.5	41.9	Harp	0

Table 17. East Schools: Leave Simultaneously, Objective: Minimize Maximum Late

All Hubs Possible

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
10	5	-3.6	43.4	Jones	3
7	12	0.8	38.2	Parson Hills	0

Hubs limited to Bayyari, Harp, and Turnbow

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
16	5	-3.5	72.7	Harp	10
10	7	-1.4	50.4	Harp	3
7	13	0.3	41.9	Harp	0

When the hubs were limited in the model, Springdale's current method was found to be the best route (fewest buses) in each instance. This validates the model's effectiveness in real situations. When all hubs were allowed, however, the model found a route that used the same number of buses but with 7.1 fewer miles and a decreased maximum lateness of two minutes when Jones is set as the hub (Figure 8). Nonetheless, Springdale considers Jones an infeasible hub location because it either does not have the necessary parking capacity or is located in an area that numerous buses would have difficultly accessing at the same time.

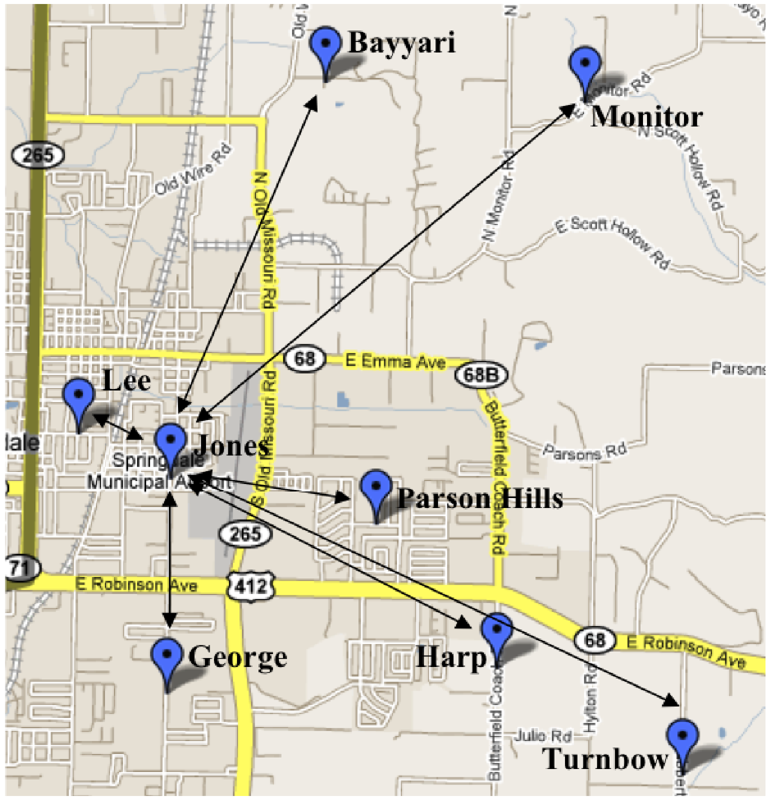


Figure 8: Springdale East Side with Jones as the Hub

The same three methods of testing the data were used for the west data, with the corresponding results being given in Tables 18 through 20.

Table 18. West Schools: Leave When Ready, Objective: Minimize Miles
All Hubs Possible

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
5	10	-8	21.5	Smith	1

Hubs limited to Hunt, Smith, Walker, and Young

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
5	10	-8	21.5	Smith	1

Table 19. West Schools: Leave When Ready, Objective: Minimize Maximum Late
All Hubs Possible

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
11	0	-9.4	41.2	Shaw	8
6	6	-8.6	27.7	Smith	3
5	10	-7.9	21.5	Smith	1

Hubs limited to Hunt, Smith, Walker, and Young

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
11	5	-9.6	41.2	Hunt	10
6	6	-8.6	27.7	Smith	3
5	10	-7.9	21.5	Smith	1

Table 20. West Schools: Leave Simultaneously, Objective: Minimize Maximum Late
All Hubs Possible

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
11	5	-9.7	41.2	Hunt	10
7	6	-8.7	33.3	Smith	5
5	11	-7.6	21.5	Smith	1

Hubs limited to Hunt, Smith, Walker, and Young

# Buses	Max Late	Avg Late	Miles	Hub	# Direct
11	5	-9.7	41.2	Hunt	10
7	6	-8.7	33.3	Smith	5
5	11	-7.6	21.5	Smith	1

The results when the hubs were limited and not limited in the west were almost identical. The only difference occurred in Table 19 where Shaw was eliminated as a possible hub. However, maximum lateness was the only factor that increased (five minutes). Smith Elementary is currently the Springdale west hub and was found most frequently to be the best hub by the model, thereby validating the model once again. However, the model requires five buses and one direct transfer (Figure 9) while Springdale currently only uses four buses on the west side and no direct pairs. This is possible because the transfer demand on the west side is considerably lower than on the east (132 students in the east versus only 35 in the west) and is achieved by using a multi-stop approach in which one bus will stop at two schools before going to the hub. A preliminary multi-stop model was developed to examine this phenomenon and indeed, we confirmed the optimality of this approach as well.

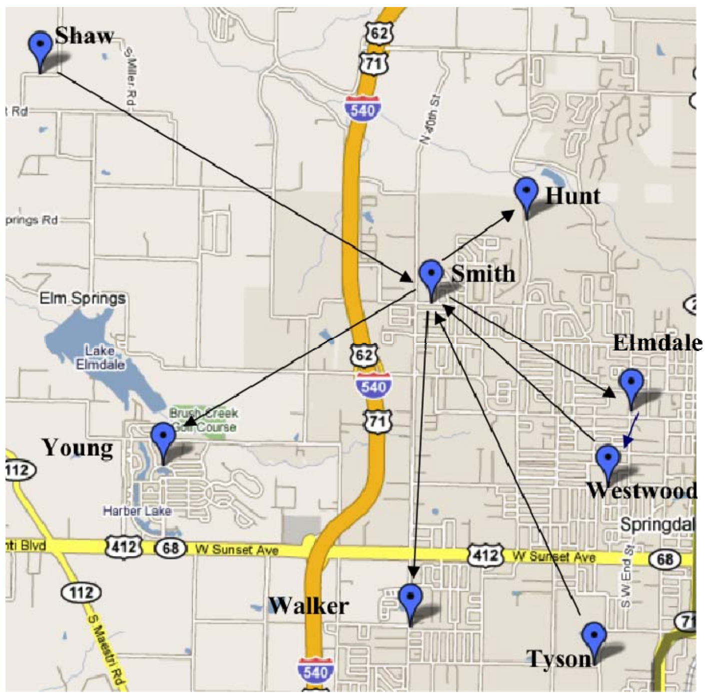


Figure 9: Springdale West Side with Smith as the Hub and one Direct Pair

4.2.2 SPS Circuit Model

When evaluating the Springdale data with the circuit model, the sixteen elementary schools were once again divided into eight east and eight west schools and analyzed separately. For each side, the model was first run with the number of buses limited to eight then to seven and so on to ultimately two buses. In both instances only two routes (using only two buses) were found, no matter what the route capacity was set to. On the east side, route two was simply the reverse of route one. Table 21 shows the two routes for the east side, miles traveled, travel time, and the bus capacity feasibility.

Table 21. East Circuit Routes and Bus Capacity Feasibility

Route 1

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
8	Turnbow	0	0	27	0	27
3	Harp	2.2	5	21	1	47
7	Parson Hills	3.6	9	3	6	44
2	George	5.6	14	10	2	52
4	Jones	6.7	17	3	16	39
5	Lee	7.4	19	3	18	24
1	Bayyari	10.6	27	4	9	19
6	Monitor	13.8	35	0	19	0

Route 2

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
6	Monitor	0	0	20	0	20
1	Bayyari	3.2	8	21	3	38
5	Lee	6.4	17	1	8	31
4	Jones	7.1	19	6	5	32
2	George	8.2	22	8	1	39
7	Parson Hills	10.2	27	5	27	17
3	Harp	11.6	31	0	9	8
8	Turnbow	13.8	35	0	8	0

The maximum number of students at any point in time is 52 on bus 1 and 39 on bus two. The two buses travel a total of 27.6 miles with a maximum time of 35 minutes. Figure 10 below shows the Springdale east side circuit routes where the black arrows represent route one and the blue arrows represent route two.

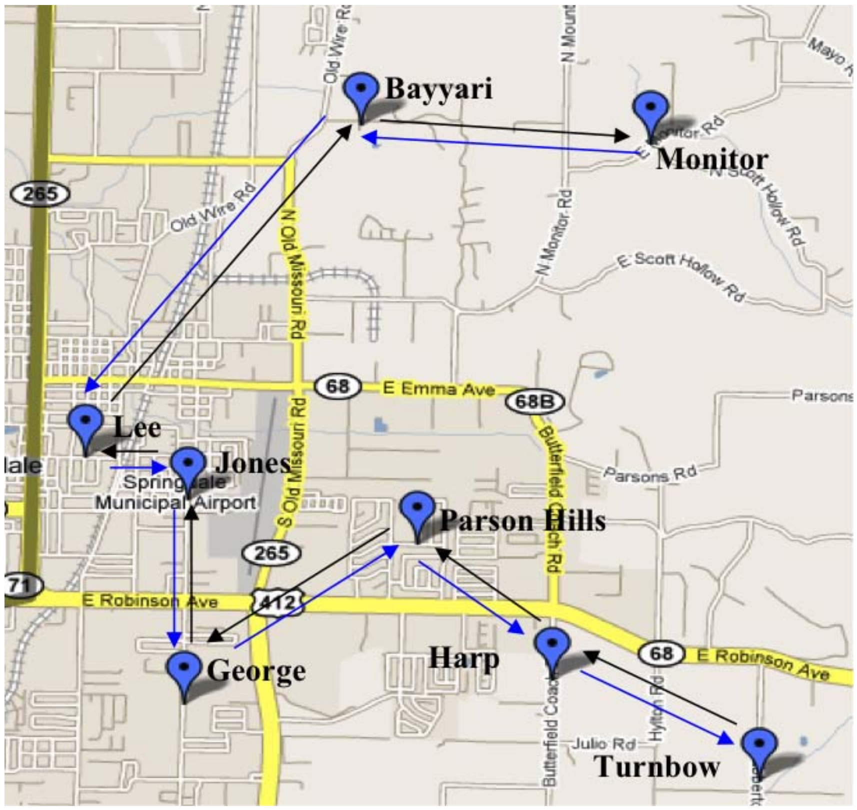


Figure 10: Springdale East Circuit Routes

On the west side only two routes were found again; however, route two only included three schools while route one visited every school. Table 22 shows the routes, miles, time traveled, and bus capacity for the west side.

Table 22. West Circuit Routes and Bus Capacity Feasibility

Route 1

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
3	Shaw	0	0	10	0	10
4	Smith	5.4	15	9	1	18
2	Hunt	6.7	18	0	5	13
1	Elmdale	8.8	24	1	3	11
7	Westwood	9.5	26	4	1	14
5	Tyson	11.1	31	5	0	19
6	Walker	12.6	35	0	7	12
8	Young	15.5	44	0	12	0

Route 2

Position	School	Miles Traveled	Travel Time (min)	# Students On	# Students Off	Current # of Students
5	Tyson	0	0	6	0	6
1	Elmdale	2	7	0	2	4
4	Smith	4.1	13	0	4	0

The maximum number of students at any point in time is 19 on bus 1 and 6 on bus two. The two buses travel a total of 19.6 miles with a maximum time of 44 minutes. Figure 11 depicts the two west routes. Again, the black arrows are route one and the blue arrows are route two.

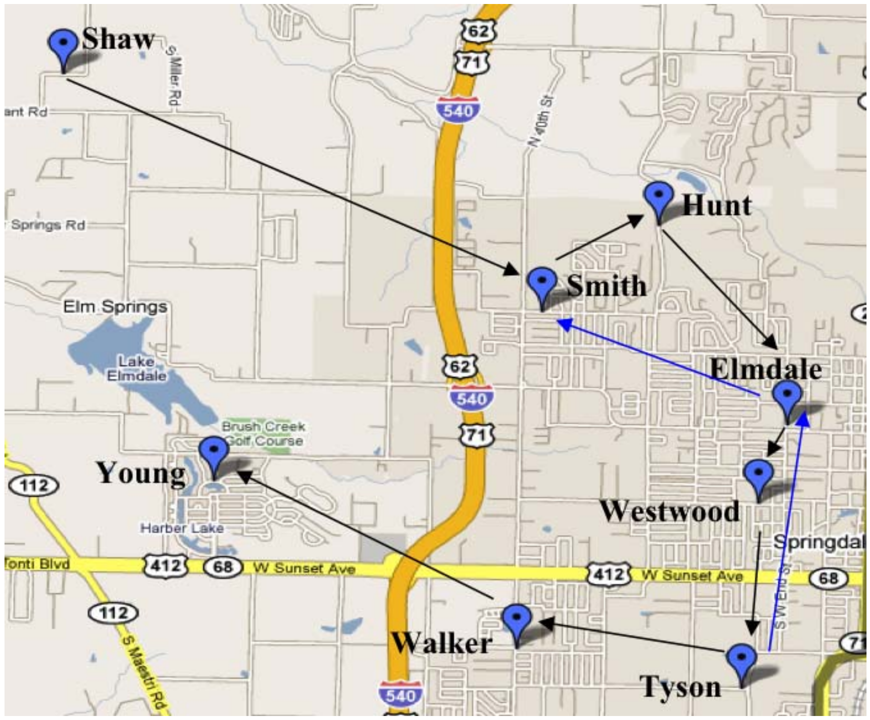


Figure 11: Springdale West Circuit Routes

The circuit routes for both the east and west sides produced lower total bus mile results than the hybrid hub-and-spoke model. These results were then presented to the Springdale Transportation Department; however, Springdale rejected the proposition. In a hub-and-spoke system, students ride a bus from their home to the school in their zone, catch a hub-and-spoke bus to the hub and then take another bus to their destination school. In a circuit model, parents must provide transportation to the school in their zone because the circuit buses must leave before the normal buses arrive due to the time required to complete the circuit. Since the majority of transfers in Springdale are mandated by the school district, the district must provide transportation for equality reasons; therefore, requiring that parents provide transportation to the base school would be met with considerable opposition. Springdale prefers to use their current

Hub-and-Spoke routing method; however, further research could further improve upon their current solution.

5 Special Needs Student Transportation

5.1 Problem Description

"Special needs" students are those children who, for various reasons, cannot attend/participate in "normal" classes for their grade level at their neighborhood school. Each special needs child has some type of disability, such as autism, speech impairment, learning disability, and/or brain injury, which prevents them from being able to learn effectively in a normal classroom environment. According to his or her specific needs, each special needs child is assigned to a designated special needs class within the school district. Special needs classes typically are designated according to the student to teacher ratio (e.g., 6:1, 10:1, etc.). The needed special needs class may or may not be located at the special needs student's neighborhood school.

School districts typically provide door-to-door transportation for all special needs children. Each special needs student is picked up at home and taken to the assigned school, regardless of how far the child's home is from the school. In our research, we assess the impact of reassigning special needs classrooms to schools based on each school's proximity to the students requiring the class's specific services and functions. Our goal is to optimize the travel experience of special needs students by minimizing the amount of bus travel required, and therefore, student bus riding times and hopefully, gasoline consumption and transportation costs.

Special needs busing is approached differently than forced transfer busing. Each student is picked up at his or her home and driven to the school providing that student's required class or service. Usually, one bus picks up all students traveling to a certain school, drops them off, and then possibly runs another route. We have met with Springdale to learn about their current special needs practices and look at their special needs busing data. In the future we plan to explore how forced transfer busing strategies can be applied to special needs busing.

5.2 Mathematical Model

As mentioned earlier, problem in special needs busing section is different than finding a circuit route or a hub. Since students are picked up from where they live and then dropped off at their school, objective becomes minimizing total distance they ride on the bus. Following assumptions are made in the model:

- Each bus has a single pick-up and drop-off location.
- Each bus has a specific capacity for students.
- At the origin point and at the destination, load on the buses should be zero.

Special needs model also has been solved with two other objective functions. Those objective functions which are described later in this section, deal with either dropping elementary students earlier or picking them up earlier.

5.2.1 Notation

Sets and Parameters

K	Set of vehicles indexed by k
P	$= \{1, \ldots, p\}$ Set of pickup locations
D	$= \{p+1, \ldots, 1\}$ Set of delivery locations
O_k	Origin depot of vehicle $k \in K$
M_k	Destination depot of vehicle $k \in K$
N	$= \{P \cup D\}$
V_k	$= \{N \cup O_k \cup M_k\}, k \in K$: Set of vertices
A_k	$= \{(i, j): i \in V, j \in V\}$: Set of arcs
c_{ij}^k	Cost of using arc $(i, j) \in A^k$ by vehicle $k \in K$
l_i	load pickup or delivered at location i $\forall i \in N$

Q_k Capacity of vehicle k

d_{ij}^k Distance travelled by vehicle $k \in K$ on arc $(i, j) \in A^k$

u_{ij} =1, if there exists load transported between location i and j, otherwise 0 $\forall (i, j) \in A^k$

Decision Variables

x_{ij}^k =1, if vehicle k use arc $(i, j) \in A^k$, $i \neq$ j, otherwise 0

y_j^k Load on vehicle $k \in K$ after picking up the load at location $j \in P$

z_i^k Distance accumulated by vehicle $k \in K$ after visiting node $i \in V_k$

5.2.2 Mixed-Integer Program

We now present three model variations, with increasing degree of reality added into them. We term the first model as OPT1. The objective of the OPT1 model as given in equation (52) is to minimize the total distance traveled by all the buses.

$$\text{Min} \quad \sum_{i,j \in N, i \neq j} \sum_{k \in K} x_{ij}^k d_{ij}^k \tag{52}$$

Constraint set (53) guarantees one and only one vehicle leaves each pickup location.

$$\sum_{j \in P \cup M_k, j \neq i} x_{ij}^k = 1 \qquad \forall i \in O_k, \forall k \in K \tag{53}$$

Constraint set (54) ensures that each pick-up location or delivery node is visited exactly once.

$$\sum_{j \in N \cup M_k, j \neq i} \sum_{k \in K} x_{ij}^k = 1 \qquad \forall i \in N \tag{54}$$

Constraint set (55) guarantees one and only one vehicle arrives to each delivery location.

$$\sum_{j \in D_k, j \neq i} x_{ij}^k = 1 \qquad \forall j \in M_k,\ \forall k \in K \tag{55}$$

Constraint set (56) ensures that if a vehicle visits a particular node (pick-up or delivery), then it leaves that node.

$$\sum_{i \in N \cup O_k, j \neq i} x_{ij}^k - \sum_{i \in N \cup M_k, j \neq i} x_{ji}^k = 0 \qquad \forall j \in N,\ \forall k \in K \tag{56}$$

Constraint sets (57) and (58) update the load on a vehicle and the distance accumulated by a vehicle after visiting a particular node.

$$y_i^k + l_j \le M\left(1 - x_{ij}^k\right) + y_j^k \qquad \forall (i, j) \in A^k, \forall k \in K, i \ne j \tag{57}$$

$$z_i^k + d_{ij}^k \le M\left(1 - x_{ij}^k\right) + z_j^k \qquad \forall (i, j) \in A^k, \forall k \in K, i \ne j \tag{58}$$

Constraints on vehicle capacity are enforced by (59) and (60).

$$l_i \le y_i^k \le Q^k \qquad \forall i \in P,\ \forall k \in K \tag{59}$$

$$0 \le y_i^k \le Q^k + l_i \qquad \forall i \in D, \forall k \in K \tag{60}$$

Constraint set (61) requires the vehicles to be empty at their initial and final locations.

$$y_i^k = 0 \qquad \forall i \in O_k \bigcup M_k \tag{61}$$

Constraint set (62) states that the vehicle has not traveled any distance when it is at its initial depot.

$$z_i^k = 0 \qquad \forall i \in O_k \tag{62}$$

Constraint set (63) ensures that node *i* is traveled before node *j* if there is demand from node *i* to node *j*.

$$\left(z_i^k + 0.1\right)u_{ij} \le z_j^k \qquad \forall (i, j) \in A^k, \forall k \in K \tag{63}$$

Constraint set (64) and (65) forces there is only one node to be visited after origin node or before destination node. Constraint set (64) and (65) forces that all vehicles have one and only one origin and one and only one destination.

$$\sum_{j \in N, i \ne j} x_{ij}^k = 1 \qquad \forall i \in O_k, \forall k \in K \tag{64}$$

$$\sum_{i \in N, i \ne j} x_{ij}^k = 1 \qquad \forall j \in M_k, \forall k \in K \tag{65}$$

Constraint set (66) is a logic constraint that defines the relation between two kinds of variables.

$$\sum_{h \in M_k} \sum_{k \in K} z_h^k = \sum_{i,j \in N, i \neq j} \sum_{k \in K} x_{ij}^k d_{ij}^k \tag{66}$$

Constraint sets (67) and (68) are variable type constraints.

$$x_{ij}^k \in \{0,1\} \qquad \forall (i,j) \in A,\ \forall k \in K \tag{67}$$

$$z_i^k \geq 0 \qquad \forall i \in V,\ \forall k \in K \tag{68}$$

OPT2 is the optimal value for the total distance when elementary school students are delivered before high school students. One parameter and one constraint are added in order to meet this new requirement. The new parameter denotes elementary schools are sequenced before high schools.

f_{ij} = 1, if node *i* should be visited before node *j*, otherwise 0, $\forall i, j \in D$

$$(z_i^k + 0.1) f_{ij} \leq z_j^k \qquad \forall i, j \in D, \forall k \in K \tag{69}$$

Constraint set (69) requires that elementary school students be delivered before high school students.

OPT3 is the optimal value of total distance when older students must be picked up after elementary school students are delivered. At this time, the bus stops for older students must be sequenced after the elementary school students. Although the same parameter and constraint for OPT2 has been added, the scale of f_{ij} has been extended as follows:

f_{ij} = 1, if node *i* should be visited before node *j*, otherwise 0, $\forall i, j \in N$

$$(z_i^k + 0.1) f_{ij} \leq z_j^k \qquad \forall i, j \in N, \forall k \in K \tag{70}$$

Constraint set (70) requires that elementary school students be picked up before high school students.

5.3 Springdale Public Schools Special Needs Case Study

Table 23. Springdale Public Schools Case Study Results

Bus NO	OPT 1	OPT2	OPT3
100	21.736	25.114	25.469
101	26.362	28.141	45.125
102	19.773	19.773	19.773
66	23.951	23.951	25.292
67	17.409	20.618	20.618
68	24.436	24.649	
79	23.523	24.684	30.519
80	25.305	27.775	32.776
95	5.638	5.638	5.638
96	27.238	27.238	
97	20.859	20.864	30.653
MD	8.411	8.411	8.411
117	12.106	12.106	12.106
118	20.408	20.408	24.922
V1	8.976	8.976	8.976
65	3.547	3.547	3.547

OPT1 is the optimal value of total distances. OPT2 is the optimal value in which elementary school students are delivered before high school students. OPT3 is the optimal value in which older students are pickup after elementary school students are delivered. The reasons for the same optimal values among OPT1, OPT2 and OPT3 are:

1) There is only one school level in the bus route.
2) The optimal value also meets the requirement of OPT2 or OPT3.
3) There is no optimal value for OPT3 because one bus stop goes to both the elementary school and the middle school and one bus stop can only be traversed once.
4) OPT3 can't be solved for buses 68 and 96.

6 Conclusions and Future Work

Forced transfer busing occurs in school districts across the country, but is a problem that is often inefficiently solved by hand. This research will continue to work with real data to hopefully find more efficient routing solutions. In the future real Fort Smith data will be collected and will be used in a hybrid model to compare with their current circuit method. Springdale's current method has been validated, but can be tested further using a multi-stop approach. However, the majority of research conducted with Springdale's data will focus on analyzing and improving Springdale's special needs busing.

The special needs student busing problem has different characteristics than the forced transfer busing problem. This problem also can be solved using a different objective function, but the main focus is still minimizing total time students spend on the bus. The results from the Springdale case study showed that analyzing problems with different objectives (focuses) will result in almost the same answers. Special needs busing is a wide open subject and needs further research and data analysis in order to make in impact in the school districts.

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